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Riemannian Acceleration in Cartesian Coordinate Based Upon the Golden Metric Tensor

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

Geometric quantities in all orthogonal curvilinear co-ordinates are built upon Euclidean geometry. This geometry is founded on a well known metric tensor called the Euclidean metric tensor. Riemannian geometry which is assumed to be more general than the Euclidean geometry was founded on an unknown metric tensor for spacetimes in gravitational fields. Therefore the Riemannian geometry itself could not be opened up for exploration and exploitation, let alone the possible application to theoretical physics. But with the discovery of a general Riemannian metric tensor called the golden metric tensor, exploitation of Riemannian geometry is now possible. We are in a position to calculate all the theoretical predictions of Riemann's geometrical and physical concepts and principles and compare them with experimental physical evidence. In this paper, we use the golden metric tensor to develop Riemannian acceleration in the Cartesian coordinate for application in theoretical physics and other related fields.

Keywords: Riemannian geometry; golden metric tensor; Riemannian acceleration; Cartesian coordinate.

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1. INTRODUCTION

Right from 1854, when the German mathematician, George Riemann published his geometry for spacetime known as Riemann geometry, it was assumed to be more general than the Euclidean geometry. It is generally accepted that Riemannian geometry has the potential of providing a more general foundation for theoretical physics [1]. However, the problem with the Riemannian geometry is that it was not founded on a known metric tensor therefore its exploitation and possible applications to theoretical physics eluded the world. Riemann during his own time (1816-1866) could not find the metric tensor(s) implicit in his geometry. Riemann therefore left behind the problem of finding the metric tensors for all gravitational fields. Einstein tried to solve this problem in his contribution to classical mechanics: Einstein's Geometrical Gravitational Field Equations [1]. It has since been believed that Einstein's Geometrical Gravitational Field Equations could lead to the construction of the metric tensors for all gravitational fields in nature. In 1916, Karl Schwarzschild introduced a metric tensor for all the gravitational field due to static homogeneous spherical distribution of mass. This metric tensor was called Schwarzschild's metric tensor. This metric tensor has been the basis for the development of Einstein's Geometrical Theory of classical mechanics in the Gravitational field known as General Relativity. In spite of the great fame since 1915, Einstein's Geometrical Gravitational Field Equations cannot be applied to generate any natural metric tensor for the gravitational fields due to any distribution of mass in nature. These equations have to be abandoned and confined to archives of history in the search for metric tensors for the gravitational fields in nature.

It is interesting to know that a metric tensor called the golden metric tensor for all gravitational fields in nature has been developed [1]. This metric tensor is valid for all four co-ordinates of spacetime and in all the regular geometries in nature and for all regular distributions of mass. In the limit of c^0, it reduces to the well known Euclidean metric tensor for all spacetimes in gravitational fields in nature, in perfect agreement with the principle of equivalence of mathematics and the principle of equivalence of physics. We are in a position to calculate all the theoretical predictions of Riemann's geometrical and physical concepts and principles and compare them with experimental physical evidence. In this paper, we use the golden metric tensor to develop Riemannian acceleration in the Cartesian coordinate for application in theoretical physics and other related fields.

2. THEORY

The spherical polar coordinate $(r, \theta, \varphi, x^0)$ are defined in terms of the Cartesian coordinates (x, y, z, x^0) by [2-4]

$$x = r sin\theta cos\varphi \tag{1}$$

$$y = r sin\theta sin\varphi \tag{2}$$

$$z = r cos\varphi \tag{3}$$

Where

$$r = (x^2 + y^2 + z^2)^{\frac{1}{2}} \tag{4}$$

$$\theta = cos^{-1}\left\{\frac{z}{(x^2+y^2+z^2)^{\frac{1}{2}}}\right\} \tag{5}$$

$$\varphi = tan^{-1}\left(\frac{y}{x}\right) \tag{6}$$

The Golden Metrix tensor for all gravitational fields in nature is given in the spherical polar coordinates $(r, \theta, \varphi, x^0)$ as [3]

$$g_{11} = \left(1 + \frac{2}{c^2}f\right)^{-1} \tag{7}$$

$$g_{22} = r^2\left(1 + \frac{2}{c^2}f\right)^{-1} \tag{8}$$

$$g_{33} = r^2 sin^2\theta\left(1 + \frac{2}{c^2}f\right)^{-1} \tag{9}$$

$$g_{00} = -\left(1 + \frac{2}{c^2}f\right) \tag{10}$$

$$g_{\mu v} = 0;\ \ otherwise \tag{11}$$

Now we have to transform this metric tensor into the cartesian coordinate system using the well known transformation relation in the theory of vector and tensor analysis given as [3,5];

$$\overline{g_{qs}} = \frac{\partial x^q}{\partial \bar{x}^q}\frac{\partial x^s}{\partial \bar{x}^s}g_{qs} \tag{12}$$

Applying the transformation relation given by (12), the metric tensor for all gravitational fields in the Cartesian coordinates are

$$g_{11} = \left(1 + \frac{2}{c^2} f\right)^{-1} \quad (13)$$

$$g_{22} = \left(1 + \frac{2}{c^2} f\right)^{-1} \quad (14)$$

$$g_{33} = \left(1 + \frac{2}{c^2} f\right)^{-1} \quad (15)$$

$$g_{00} = -\left(1 + \frac{2}{c^2} f\right) \quad (16)$$

$$g_{\mu v} = 0; \;\; otherwise \quad (17)$$

Therefore, the contravariant tensor of the metrc tensor is given as;

$$g^{11} = \left(1 + \frac{2}{c^2} f\right) \quad (18)$$

$$g^{22} = \left(1 + \frac{2}{c^2} f\right) \quad (19)$$

$$g^{33} = \left(1 + \frac{2}{c^2} f\right) \quad (20)$$

$$g^{00} = -\left(1 + \frac{2}{c^2} f\right)^{-1} \quad (21)$$

$$g_{\mu v} = 0; otherwise$$

Also from our knowledge of vector and tensor analysis, the Riemannian linear velocity is given by

$$(U_R)_{x^0} = (g_{00})^{\frac{1}{2}} \dot{x}^0 \quad (22)$$

$$(U_R)_x = (g_{11})^{\frac{1}{2}} \dot{x} \quad (23)$$

$$(U_R)_y = (g_{22})^{\frac{1}{2}} \dot{y} \quad (24)$$

$$(U_R)_z = (g_{33})^{\frac{1}{2}} \dot{z} \quad (25)$$

And explicitly, taking x^0=ct we can express this velocity as

$$(U_R)_{x^0} = -c\left(1 + \frac{2}{c^2} f\right)^{\frac{1}{2}} \dot{t} \quad (26)$$

$$(U_R)_x = \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \dot{x} \quad (27)$$

$$(U_R)_y = \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \dot{y} \quad (28)$$

$$(U_R)_z = \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \dot{z} \quad (29)$$

2.1 The Golden Riemannian Linear Acceleration Tensor

We want to recall the definition of the linear acceleration tensor for all gravitational fields in nature and show how to express them in terms of the Golden metric tensor in the Cartesian coordinate. According to the theory of tensor analysis the linear acceleration tensor in 4-dimensional spacetime, a_R^μ , is given in all gravitational fields and all orthogonal curvilinear coordinates x^μ by

$$a^\mu = \ddot{x}^\mu + \Gamma^\mu_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta \quad (30)$$

Where $\Gamma^\mu_{\alpha\beta}$ are the Christoffel symbols of the second kind pseudo tensor and a dot denotes one differentiation with respect to proper time τ. This quantity is called the Riemannain linear acceleration tensor in all 4-dimensions in orthogonal curvilinear coordinates and all gravitational fields based upon the golden metric tensor for all gravitational fields in nature. $\Gamma^\mu_{\alpha\beta}$ is defined in all gravitational fields and all orthogonal curvilinear coordinates as

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\epsilon}\left(g_{\alpha\epsilon,\beta} + g_{\epsilon\beta,\alpha} - g_{\alpha\beta,\epsilon}\right) \quad (31)$$

Where $g_{\alpha\epsilon}$ is the metric tensor and a comma denotes one partial differentiation with respect to the Cartesian coordinate. Thus, the non-zero terms are given by

$$\Gamma^0_{00} = \frac{1}{2} g^{00}\left(g_{00,0} + g_{00,0} - g_{00,0}\right) \quad (32)$$

$$= \frac{1}{2} g^{00} g_{00,0}$$
$$= \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,0 \quad (33)$$

$$\Gamma^0_{01} = \frac{1}{2} g^{00}\left(g_{00,1} + g_{01,0} - g_{01,0}\right) \quad (34)$$
$$= \frac{1}{2} g^{00} g_{00,1}$$

$$= \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,1 \quad (35)$$

$$\Gamma^0_{01} = \Gamma^0_{10}$$

$$\Gamma^0_{02} = \frac{1}{2} g^{00}\left(g_{00,2} + g_{02,0} - g_{02,0}\right) \quad (36)$$

$$= \frac{1}{2} g^{00} g_{00,2}$$

$$= \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,2 \quad (37)$$

$$\Gamma^0_{02} = \Gamma^0_{20}$$
$$\Gamma^0_{03} = \frac{1}{2} g^{00}\left(g_{00,3} + g_{03,0} - g_{03,0}\right) \quad (38)$$

$$= \frac{1}{2} g^{00} g_{00,3}$$
$$= \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,3 \quad (39)$$

$$\Gamma^0_{03} = \Gamma^0_{30}$$
$$\Gamma^0_{22} = \frac{1}{2} g^{00}\left(g_{02,2} + g_{02,2} - g_{22,0}\right) \quad (40)$$

$$= -\frac{1}{2} g^{00} g_{22,0}$$
$$= -\frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-3} f,0 \quad (41)$$

$$\Gamma^0_{11} = \frac{1}{2} g^{00}\left(g_{01,1} + g_{01,1} - g_{11,0}\right) \quad (42)$$

$$= -\frac{1}{2} g^{00} g_{11,0}$$
$$= -\frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-3} f,0 \quad (43)$$

$$\Gamma^0_{33} = \frac{1}{2} g^{00}\left(g_{03,3} + g_{03,3} - g_{33,0}\right) \quad (44)$$

$$= -\frac{1}{2} g^{00} g_{33,0}$$

$$= -\frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-3} f,0 \quad (45)$$

$$\Gamma^1_{00} = \frac{1}{2} g^{11}\left(g_{01,0} + g_{10,0} - g_{00,1}\right) \quad (46)$$

$$= -\frac{1}{2} g^{11} g_{00,1}$$

$$= \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right) f,1 \quad (47)$$

$$\Gamma^1_{11} = \frac{1}{2} g^{11}\left(g_{11,1} + g_{11,1} - g_{11,1}\right)$$
$$= -\frac{1}{2} g^{11} g_{11,1}$$
$$= -\frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,1 \quad (48)$$

$$\Gamma^1_{12} = \frac{1}{2} g^{11}\left(g_{11,2} + g_{12,1} - g_{12,1}\right) \quad (49)$$

$$= \frac{1}{2} g^{11} g_{11,2}$$

$$= -\frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,2 \quad (50)$$

$$\Gamma^1_{13} = \frac{1}{2} g^{11}\left(g_{11,3} + g_{13,1} - g_{13,1}\right)$$
$$= \frac{1}{2} g^{11} g_{11,3}$$

$$= -\frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,3 \quad (51)$$

$$\Gamma^1_{10} = \frac{1}{2} g^{11}\left(g_{11,0} + g_{10,1} - g_{10,1}\right) \quad (52)$$

$$= \frac{1}{2} g^{11} g_{11,0}$$

$$= -\frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,0 \quad (53)$$

$$\Gamma^1_{22} = \frac{1}{2} g^{11}\left(g_{12,2} + g_{12,2} - g_{22,2}\right) \quad (54)$$

$$= -\frac{1}{2} g^{11} g_{22,1}$$

$$= \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,1 \quad (55)$$

$$\Gamma^1_{33} = \frac{1}{2} g^{11}\left(g_{13,3} + g_{13,3} - g_{33,1}\right) \quad (56)$$

$$= -\frac{1}{2} g^{11} g_{33,1}$$

$$= \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,1 \quad (57)$$

$$\Gamma^2_{00} = \frac{1}{2} g^{22}\left(g_{20,0} + g_{20,0} - g_{00,2}\right) \quad (58)$$

$$= -\frac{1}{2} g^{22} g_{00,2}$$

$$= \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right) f,2 \quad (59)$$

$$\Gamma^2_{11} = \frac{1}{2} g^{22}\left(g_{21,2} + g_{21,2} - g_{11,2}\right) \quad (60)$$

$$= -\frac{1}{2} g^{22} g_{11,2}$$

$$= \frac{1}{c^2}\left(1 + \frac{2}{c^2} f\right)^{-1} f,2 \quad (61)$$

$$\Gamma^2_{12} = \frac{1}{2} g^{22}\left(g_{21,2} + g_{22,1} - g_{12,2}\right) \quad (62)$$
$$= \frac{1}{2} g^{22} g_{22,1}$$

$$= -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,1 \qquad (63)$$

$$\Gamma_{23}^{2} = \frac{1}{2}g^{22}(g_{22,3}+g_{23,2}-g_{23,2}) \qquad (64)$$

$$= \frac{1}{2}g^{22}g_{22,3}$$

$$= -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,3 \qquad (65)$$

$$\Gamma_{22}^{2} = \frac{1}{2}g^{22}(g_{22,2}+g_{22,2}-g_{22,2}) \qquad (66)$$

$$= \frac{1}{2}g^{22}g_{22,2}$$

$$= -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,2 \qquad (67)$$

$$\Gamma_{33}^{2} = \frac{1}{2}g^{22}(g_{23,3}+g_{23,3}-g_{33,2}) \qquad (68)$$

$$= -\frac{1}{2}g^{22}g_{33,2}$$

$$= \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,2 \qquad (69)$$

$$\Gamma_{00}^{3} = \frac{1}{2}g^{33}(g_{30,0}+g_{30,0}-g_{00,3}) \qquad (70)$$

$$= -\frac{1}{2}g^{33}g_{00,3}$$

$$= \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right) f,3 \qquad (71)$$

$$\Gamma_{11}^{3} = \frac{1}{2}g^{33}(g_{31,1}+g_{31,1}-g_{11,3}) \qquad (72)$$

$$= -\frac{1}{2}g^{33}g_{11,3}$$

$$= \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,3 \qquad (73)$$

$$\Gamma_{22}^{3} = \frac{1}{2}g^{33}(g_{32,2}+g_{32,2}-g_{22,3}) \qquad (74)$$

$$= -\frac{1}{2}g^{33}g_{22,3}$$

$$= \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,3 \qquad (75)$$

$$\Gamma_{33}^{3} = \frac{1}{2}g^{33}(g_{33,3}+g_{33,3}-g_{33,3}) \qquad (76)$$

$$= \frac{1}{2}g^{33}g_{33,3}$$

$$= -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,3 \qquad (77)$$

$$\Gamma_{23}^{3} = \frac{1}{2}g^{33}(g_{32,3}+g_{33,2}-g_{23,3}) \qquad (78)$$

$$= \frac{1}{2}g^{33}g_{33,2}$$

$$= -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,2 \qquad (79)$$

$$\Gamma_{31}^{3} = \frac{1}{2}g^{33}(g_{33,1}+g_{31,3}-g_{31,3}) \qquad (80)$$

$$= \frac{1}{2}g^{33}g_{33,1}$$

$$= -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,1 \qquad (81)$$

$$\Gamma_{30}^{3} = \frac{1}{2}g^{33}(g_{33,0}+g_{30,3}-g_{30,3})$$

$$= \frac{1}{2}g^{33}g_{33,0}$$

$$= -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,0 \qquad (82)$$

$$\Gamma_{02}^{2} = \frac{1}{2}g^{22}(g_{20,2}+g_{22,0}-g_{02,2}) \qquad (83)$$

$$= \frac{1}{2}g^{22}g_{22,0}$$

$$= -\frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,0 \qquad (84)$$

Hence, applying (32)-(84) in (31) we obtain the golden Riemannain acceleration vector in the Cartesian coordinate as

$$a_R^0 = \ddot{x}^0 + \Gamma_{\alpha\beta}^{0}\dot{x}^{\alpha}\dot{x}^{\beta} \qquad (85)$$

$$= \ddot{x}^0 + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}(\dot{x}^0)^2 + \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\dot{x}^0\dot{x} + \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\dot{x}^0\dot{y} + \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\dot{x}^0\dot{z} - \left(1+\frac{2}{c^2}f\right)^{-3} f_{,0}(\dot{x})^2 - \left(1+\frac{2}{c^2}f\right)^{-3} f_{,0}(\dot{y})^2 - \left(1+\frac{2}{c^2}f\right)^{-3} f_{,0}(\dot{z})^2 \quad (86)$$

$$a_R^1 = \ddot{x}^1 + \Gamma^1_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = \ddot{x} + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right) f_{,1}(\dot{x}^0)^2 - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}\dot{x}^0\dot{x} - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}(\dot{x})^2 - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\dot{x}\dot{y} - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}\dot{x}\dot{z} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}(\dot{y})^2 - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}(\dot{z})^2 \quad (87)$$

$$a_R^2 = \ddot{x}^2 + \Gamma^2_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = \ddot{y} + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right) f_{,2}(\dot{x}^0)^2 + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}(\dot{x})^2 - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\dot{x}\dot{y} - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}\dot{y}\dot{z} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}(\dot{y})^2 + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}(\dot{z})^2 \quad (88)$$

$$a_R^3 = \ddot{x}^3 + \Gamma^3_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = \ddot{z} + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right) f_{,3}(\dot{x}^0)^2 + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}(\dot{x})^2 + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}(\dot{y})^2 - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3}(\dot{z})^2 - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2}\dot{y}\dot{z} - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\dot{x}\dot{z} - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0}\dot{x}^0\dot{z} \quad (89)$$

2.2 The Golden Riemannian Linear Acceleration Vector

The golden Riemannian linear acceleration tensor have corresponding Riemannian linear acceleration vector given as

$$(a_R)_x = (g_{11})^{\frac{1}{2}} a_R^1 \quad (90)$$

$$(a_R)_y = (g_{22})^{\frac{1}{2}} a_R^2 \quad (91)$$

$$(a_R)_z = (g_{33})^{\frac{1}{2}} a_R^3 \quad (92)$$

$$(a_R)_{x^0} = (g_{00})^{\frac{1}{2}} a_R^0 \quad (93)$$

Given $x^0 = ct$ we express the golden Riemannian acceleration vector as follows;

$$(a_R)_x = \ddot{x}\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} + \left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} f_{,1}(\dot{t})^2 - \frac{2}{c}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,0}\,\dot{x}\dot{t} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,1}(\dot{x})^2 - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,2}\,\dot{x}\dot{y} - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,3}\,\dot{x}\dot{z} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,1}(\dot{y})^2 - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,1}(\dot{z})^2 \quad (94)$$

$$(a_R)_y = \left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\ddot{y} + \left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} f_{,2}(\dot{t})^2 + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,2}(\dot{x})^2 - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,1}\,\dot{x}\dot{y} - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,3}\,\dot{y}\dot{z} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,2}(\dot{y})^2 + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,2}(\dot{z})^2 \quad (95)$$

$$(a_R)_z = \left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\ddot{z} + \left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} f_{,3}(\dot{t})^2 + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,3}(\dot{x})^2 + \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,3}(\dot{y})^2 - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,3}(\dot{z})^2 - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,2}\dot{y}\dot{z} - \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,1}\dot{x}\dot{z} - \frac{2}{c}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,0}\dot{t}\dot{z} \quad (96)$$

$$(a_R)_{x^0} = c\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\ddot{t} + \left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,0}(\dot{t})^2 + \frac{2}{c}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,1}\dot{x}\dot{t} + \frac{2}{c}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,2}\dot{t}\dot{y} + \frac{2}{c}\left(1+\frac{2}{c^2}f\right)^{-\frac{3}{2}} f_{,2}\dot{t}\dot{z} - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{7}{2}} f_{,0}(\dot{x})^2 - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{7}{2}} f_{,0}(\dot{y})^2 - \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-\frac{7}{2}} f_{,0}(\dot{z})^2 \quad (97)$$

3. RESULTS AND DISCUSSION

Equations (26), (27), (28) and (29) are the Riemannian velocity in the cartesian coordinate system using the golden metric tensor while (94), (95), (96) and (97) are the Riemannian acceleration in the cartesian coordinate using the golden metric tensor. These results are mathematically most elegant, physically most natural and satisfactory for describing the motion of particles of nonzero rest masses and its consequences in the cartesian coordinate. They also correspond to a generalization of the equations of motion and all the fundamental quantities in the cartesian coordinate.

4. CONCLUSION

It is most interesting and instructive to note that the Riemannian velocity and Riemannian acceleration obtained reduces, in the limit of c^0 to the pure Euclidean results and otherwise contain post Euclidean corrections of all orders of c^{-2}.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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