



## Exact Solution for the Zakhrov-Kuzentsov Equation by Adomian Decomposition Method

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### Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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## Abstract

The Zakharov-kuznetsov equation (ZK-equation) governs the behavior of weakly nonlinear ion-acoustic waves in plasma comprising cold ions and not isothermal electrons in the presence of a uniform magnetic field. This equation is a nonlinear equation. The main objective in this paper is to find an exact solution of ZK-equation using Adomian decomposition method. An exact solution of ZK(n,n,n) is derived by Adomian decomposition method. The solution of types ZK(2,2,2) and ZK(3,3,3) are presented in many examples to show the ability and efficiency of the method for ZK-equation. The solution is calculated in the form of convergent power series with easily computable components.

*Keywords:* Zakhrov-Kuzentsov equation; Adomain decomposition method; analytical solution; Nonlinear partial differential equations.

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## 1 Introduction

The theoretical studies on soliton formation and its propagation in plasmas with multi-component magnetized plasma led to the derivation of special nonlinear wave equation by V.E. Zakharov and E. A. Kuznetsov (1974), [1]. This equation known after that by Zakharov-kuznetsov equation (ZK-equation). The ZK-equation governs the behavior of weakly nonlinear ion-acoustic waves in plasma comprising cold ions and not isothermal electrons in the presence of a uniform magnetic field [2, 3]. The equation under consideration is a two-dimensional version of the ZK-equation that is

$$u_t + u\partial_x u + \partial_x \Delta u = 0. \quad (1.1)$$

In non-dimensional form, the ion dynamics are governed by the following system of the equations:

$$\begin{cases} N_t + \nabla \cdot (nv) = 0, \\ V_t + (v \cdot \nabla)V = -\nabla\phi + V \times (\Omega \hat{X}), \\ \nabla^2 \phi = n_e - n, \end{cases} \quad (1.2)$$

where  $n, n_e, V, \phi$  and  $\Omega$  are respectively the non-dimensional ion number density, the electron number density, the ion fluid velocity, the electrostatic potential and the external magnetic field whose direction is specified by unit vector  $\hat{X}$  [4].

The Adomian decomposition method (ADM) is proposed by George Adomian 1994. This method has been proved to be a reliable and efficient for a wide class of differential, integral and integro-differential equations of linear and nonlinear models [5, 6, 7]. The method has been applied in a straight forward manner to homogeneous, inhomogeneous, linear and nonlinear problems without any restrictive assumptions, linearization or perturbation. The convergence analysis of the series of ADM has been proved. Exact solutions can be achieved by the known form of the series solution.

Many methods have been developed to solve ZK-equation, such as employment of the hyperbolic type method is used for finding the solution features in relation to laboratory and space plasma environments. Where this method has been unsuccessful, an alternate method has been developed to yield the solution propagation. The features of the nonlinear plasmaacoustic waves, which depend on the plasma composition, affect the coexistence of compressive and rare active solitary waves. A new formalism is used for finding the solution propagation from the nonlinear wave equation with strong, as well as weak nonlinearity [8]. The nonlinear quantum ion acoustic waves propagation obliquely in two dimensions in super dens, magnetized electron positron-ion quantum plasma investigated on the basis of quantum hydrodynamic model [9]. An exact travelling wave solution for the generalized (2+1) dimensional ZK-MEW equation was constructed, by using the solution of auxiliary ODE given by sirendaoreji. Alaattin Esen and Selcuk Kutluay used this method to solve ZK-equation [10, 11].

Homotopy perturbation method (HPM) and variational iteration method (VIM) are implemented for solving the a nonlinear dispersive ZK(3,3,3). Result reveal that the HPM and VIM are very effective, convenient and accurate to system of nonlinear partial differential equations [3]. The differential transform method (DTM) is applied in finding an approximate analytical solution of ZK-equation. Two special cases, ZK(2,2,2) and ZK(3,3,3) are chosen to illustrate the powerful of the method. The DTM is an efficient technique for finding solutions without the need of a linearization process and the solution is found in the form of a rapidly convergent series. But the computational operations of the components are complicated to some extent [12].

However, there are no results on Zakhrov-Kuznetsov Equation by Adomian Decomposition Method.

This paper is organized as follows. In section 2, we recall some basic concepts and properties for Adomian Decomposition Method (ADM). In section 3, we provide the derivation for the solution of

ZK (n, n, n). In section 4, we solve some numerical examples of ZK Equation. In the last section, we discussion and conclusion for about this paper.

## 2 Basic concepts of the Adomian Decomposition Method

We recall some basic concepts and properties for Adomian Decomposition Method (ADM). The ADM has been applied to a wide class of linear partial differential equations. The method has been applied direct and in a straight forward manner to homogeneous and in homogeneous problems without any restrictive assumption or linearization. The method usually decomposes the unknown function  $u$  into an infinite sum of components that will be determined recursively through iterations [13].

In this section we The Adomian decomposition method will be applied in this paper to handle non-linear partial deferential equations. An important remark should be made here concerning the representation of the non-linear terms that appear in the equation. Although the linear term  $u$  is expressed as an infinite series of components, the Adomian is expressed as an infinite series of components, the Adomian decomposition method requires a special representation for the non-linear terms such as  $u, u^2, u^3, u^4, \sin u, e^u, uu_x, u_x^2, \dots$ etc. that appear in the equation [14].

The method introduces a formal algorithm to establish a proper representation for all forms of non-linear terms. The representation of the non-linear terms is necessary to handle the non-linear equations in an effective and successful way. An alternative algorithm for calculating Adomian polynomials will be outlined in details supported by illustrative examples[15].

The Adomian decomposition method suggests that the un-known linear function  $U$  be represented by the decomposition series  $u = \sum_{n=0}^{\infty} u_n$ , where the components  $u_n (n \geq 0)$  can be elegantly computed in recursive way [16]. However, the non linear term  $N(u)$ , such as  $u^2, u^3, u^4 \dots$ etc. can be expressed by an infinite series of the so called Adomian Polynomials (Adp) given in the form

$$N(u) = \sum_{n=0}^{\infty} (Adp)_n(u_0, u_1, u_2, u_3, \dots, u_n), \quad (2.1)$$

There the so-called Adomain polynomial  $(Adp)_n$  can be evaluated for all forms of nonlinearity. Several schemes have been introduced in the literature by researchers to calculate Adomian polynomials. Adomian introduced a scheme for the calculation of Adomian polynomials that was formally justified. An alternative reliable method that is based on algebraic and trigonometric identities and on Taylor series has been developed. The alternative method employs only elementary operations and not require specific formulas [17].

The Adomian polynomials  $(Adp)_n$  for the nonlinear term  $N(u)$  can be evaluated by using the following expression [18, 19].

$$(Adp)_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} [N(\sum_{j=0}^n \lambda^j u_j)]_{\lambda=0}. \quad n = 0, 1, 2, 3, \dots \dots \quad (2.2)$$

The general formula (2.2) can be simplified as follows. Assuming that the nonlinear function is  $N(u)$  therefore by using (2.2) Adomian polynomials are given by:

$$\left\{ \begin{array}{l} (Adp)_0 = N(u_0), \\ (Adp)_1 = u_1 N'(u_0), \\ (Adp)_2 = u_2 N'(u_0) + \frac{1}{2!} u_1^2 N''(u_0), \\ (Adp)_3 = u_3 N'(u_0) + u_1 u_2 N''(u_0) + \frac{1}{3!} u_1^3 N'''(u_0), \\ (Adp)_4 = u_4 N'(u_0) + \left(\frac{1}{2!} u_2^2 + u_1 u_3\right) N''(u_0) + \frac{1}{2!} u_1^2 u_2 N'''(u_0) + \frac{1}{4!} u_1^4 N^{(4)}(u_0), \\ \cdot \\ \cdot \\ \cdot \end{array} \right. \quad (2.3)$$

other polynomials can be generated in a similar manner in [5].

### 3 Derivation the Solution of ZK (n, n, n)

In this section, we provide the Derivation for the solution of ZK (n, n, n).

The general formof the ZK-equation is given by:

$$u_t + a(u^n)_x + b(u^n)_{3x} + c(u^n)_{yyx} = 0; \quad n > 1, \quad (3.1)$$

the coefficients  $a, b, c$  are constants.

$$L_t u = -a(u^n)_x - b(u^n)_{3x} - c(u^n)_{yyx}, \quad (3.2)$$

where  $L_t = \frac{\partial}{\partial t}$ ;  $L_t^{-1} = \int_0^t (\dots) dt$ .

Now, we applying  $L_t^{-1}$  in to both side of (3.2), we obtain that

$$u(x, y, t) = u(x, y, 0) - aL_t^{-1}\left(\frac{\partial}{\partial x}(N)\right) - bL_t^{-1}\left(\frac{\partial^3}{\partial x^3}(N)\right) - cL_t^{-1}\left(\frac{\partial^3}{\partial y^2 \partial x}(N)\right), \quad (3.3)$$

where  $N \equiv u^n$ ;  $u(x, y, 0) = u_0(x, y, t)$  is the initial condition.

Now let

$$\left\{ \begin{array}{l} u(x, y, t) = \sum_{n=0}^{\infty} u_n(x, y, t), \\ \frac{\partial}{\partial x}(N) = \sum_{n=0}^{\infty} A_n, \\ \frac{\partial^3}{\partial x^3}(N) = \sum_{n=0}^{\infty} B_n, \\ \frac{\partial^3}{\partial y^2 \partial x}(N) = \sum_{n=0}^{\infty} C_n, \end{array} \right. \quad (3.4)$$

where  $A_n, B_n, C_n$  are called Adomian polynomial it is given by:

$$\begin{cases} A_n = \frac{\partial}{\partial x} \left( \frac{1}{n!} \frac{d^n}{d\lambda^n} \left( N \left( \sum_{j=0}^n \lambda^j u_j \right)_{\lambda=0} \right) \right), & n = 0, 1, 2, 3, \dots \\ B_n = \frac{\partial^3}{\partial x^3} \left( \frac{1}{n!} \frac{d^n}{d\lambda^n} \left( N \left( \sum_{j=0}^n \lambda^j u_j \right)_{\lambda=0} \right) \right), & n = 0, 1, 2, 3, \dots \\ C_n = \frac{\partial^3}{\partial y^2 \partial x} \left( \frac{1}{n!} \frac{d^n}{d\lambda^n} \left( N \left( \sum_{j=0}^n \lambda^j u_j \right)_{\lambda=0} \right) \right). & n = 0, 1, 2, 3, \dots \end{cases} \quad (3.5)$$

Then inserting (3.4) and (3.5) into (3.3), we see that

$$\sum_{n=0}^{\infty} u_n(x, y, t) = u(x, y, 0) - L_t^{-1} \left( a \sum_{n=0}^{\infty} A_n + b \sum_{n=0}^{\infty} B_n + c \sum_{n=0}^{\infty} C_n \right). \quad (3.6)$$

Hence, we can rewrite (3.6) as

$$\sum_{n=0}^{\infty} u_{n+1}(x, y, t) = -L_t^{-1} \left( a \sum_{n=0}^{\infty} A_n + b \sum_{n=0}^{\infty} B_n + c \sum_{n=0}^{\infty} C_n \right). \quad (3.7)$$

Now, we write

$$\begin{cases} u_0(x, y, t) = u(x, y, 0), & n = 0, \\ u_{n+1}(x, y, t) = -L_t(aA_n + bB_n + cC_n). & n \geq 0 \end{cases}$$

Then the general solution is obtained by

$$u(x, y, t) = \sum_{n=0}^{\infty} u_n(x, y, t). \quad (3.8)$$

## 4 Main Results

In this section, we find an exact solution of ZK equation types ZK(2,2,2) and ZK(3,3,3), using by Adomain decomposition method.

**Example 4.1** Solve the equation

$$u_t + (u^2)_x + (u^2)_{3x} = 0, \quad (4.1)$$

the initial condition:

$$u_0(x, t) = u(x, 0) = \frac{4c}{3} \cos^2 \frac{x}{4}, \quad c \text{ is constant.} \quad (4.2)$$

**Solution:**

$$L_t u = -((u^2)_x + (u^2)_{3x}), \quad \text{where } L_t = \frac{\partial}{\partial t}, \quad L_t^{-1} = \int_0^t (\dots) dt. \quad (4.3)$$

We applying  $L_t^{-1}$  to both sides in (4.3), we get

$$u(x, t) = u(x, 0) - \int_0^t (A_n + B_n) dt, \quad (4.4)$$

where

$$u_0(x, t) = \frac{2c}{3} \left( \cos \frac{x}{2} + 1 \right). \quad (4.5)$$

From (4.4), we obtain that

$$u_{n+1}(x, t) = - \int_0^t (A_n + B_n) dt, \tag{4.6}$$

by using (3.5) and (4.6), its easy to calculate that

$$\left\{ \begin{array}{l} u_1 = \frac{c^2 t}{3} \sin \frac{x}{2}, \\ u_2 = -\frac{c^3 t^2}{12} \cos \frac{x}{2}, \\ u_3 = -\frac{1}{72} c^4 t^3 \sin \frac{x}{2}, \\ u_4 = \frac{1}{576} c^5 t^4 \cos \frac{x}{2}, \\ u_5 = \frac{1}{5760} c^6 t^5 \sin \frac{x}{2}. \\ \cdot \\ \cdot \\ \cdot \end{array} \right. \tag{4.7}$$

From (4.5) and (4.7), the solution of the (4.1)its given by

$$\begin{aligned} u(x, t) &= u_0(x, t) + u_1(x, t) + u_2(x, t) + u_3(x, t) + u_4(x, t) + u_5(x, t) + \dots \\ &= \frac{2c}{3} + \frac{2c}{3} \cos \frac{x}{2} \left( 1 - \frac{c^2 t^2}{2^2 2!} + \frac{c^4 t^4}{2^4 4!} - \frac{c^6 t^6}{2^6 6!} + \dots \right) \\ &+ \frac{2c}{3} \sin \frac{x}{2} \left( \frac{ct}{2} - \frac{c^3 t^3}{2^3 3!} + \frac{c^5 t^5}{2^5 5!} - \frac{c^7 t^7}{2^7 7!} + \dots \right), \\ &= \frac{2c}{3} + \frac{2c}{3} \left( \cos \frac{x}{2} \cos \frac{ct}{2} + \sin \frac{x}{2} \sin \frac{ct}{2} \right), \\ &= \frac{2c}{3} + \frac{2c}{3} \cos \left( \frac{x-ct}{2} \right), \\ &= \frac{2c}{3} \left( 1 + \cos \left( \frac{x-ct}{2} \right) \right), \\ &= \frac{4c}{3} \cos^2 \left( \frac{x-ct}{4} \right). \end{aligned} \tag{4.8}$$

**Example 4.2** Solve the equation

$$u_t - (u^2)_x + \frac{1}{8}(u^2)_{3x} + \frac{1}{8}(u^2)_{yyx} = 0, \tag{4.9}$$

the initial condition:

$$u_0(x, y, t) = u(x, y, 0) = \frac{4}{3} \lambda \sinh^2 \left( \frac{x+y}{2} \right), \quad \lambda \text{ is constant.} \tag{4.10}$$

**Solution:**

$$L_t u = (u^2)_x - \frac{1}{8}(u^2)_{3x} - \frac{1}{8}(u^2)_{yyx}, \tag{4.11}$$

by using  $L_t^{-1}$  to both side into (4.11),we obtain that

$$u(x, y, t) = \frac{4\lambda}{3} \sinh^2 \left( \frac{x+y}{2} \right) + \int_0^t \left( A_n - \frac{1}{8} B_n - \frac{1}{8} C_n \right) dt. \tag{4.12}$$

From (4.12) we have

$$u_0 = \frac{2\lambda}{3}(\cosh(x+y) - 1), \tag{4.13}$$

and

$$u_{n+1} = \int_0^t (A_n - \frac{1}{4}B_n)dt, \tag{4.14}$$

where,  $B_n = C_n$ .

By using (3.5) and (4.14), its easy to calculate that

$$\left\{ \begin{array}{l} u_1 = -\frac{2}{3}\lambda^2 t \sinh(x+y), \\ u_2 = \frac{1}{3}\lambda^3 t^2 \cosh(x+y), \\ u_3 = -\frac{1}{9}\lambda^4 t^3 \sinh(x+y), \\ u_4 = \frac{1}{36}\lambda^5 t^4 \cosh(x+y), \\ u_5 = -\frac{1}{180}\lambda^6 t^5 \sinh(x+y). \\ \cdot \\ \cdot \\ \cdot \end{array} \right. \tag{4.15}$$

From (4.13) and (4.15), the solution of the (4.9) its given by

$$\begin{aligned} u(x, y, t) &= u_0(x, y, t) + u_1(x, y, t) + u_2(x, y, t) + u_3(x, y, t) + u_4(x, y, t) + u_5(x, y, t) + \dots \\ &= -\frac{2\lambda}{3} + \frac{2\lambda}{3} \cosh(x+y) \left(1 + \frac{\lambda^2 t^2}{2!} + \frac{\lambda^4 t^4}{4!} + \frac{\lambda^6 t^6}{6!} + \dots\right) \\ &\quad - \frac{2\lambda}{3} \sinh(x+y) \left(\lambda t + \frac{\lambda^3 t^3}{3!} + \frac{\lambda^5 t^5}{5!} + \frac{\lambda^7 t^7}{7!} + \dots\right), \\ &= -\frac{2\lambda}{3} + \frac{2\lambda}{3} (\cosh(x+y) \cosh(\lambda t) - \sinh(x+y) \sinh(\lambda t)), \\ &= \frac{2\lambda}{3} (\cosh(x+y - \lambda t) - 1), \\ &= \frac{4\lambda}{3} \sinh^2\left(\frac{x+y - \lambda t}{2}\right). \end{aligned} \tag{4.16}$$

**Example 4.3** Solve the equation

$$u_t - (u^3)_x + 2(u^3)_{3x} + 2(u^3)_{yyx} = 0, \tag{4.17}$$

the initial condition:

$$u_0(x, y, t) = u(x, y, 0) = \sqrt{\frac{3\lambda}{2}} \sinh\left(\frac{1}{6}(x+y)\right), \quad \lambda \text{ is constant.} \tag{4.18}$$

**Solution:**

$$L_t u = (u^3)_x - 2(u^3)_{3x} - 2(u^3)_{yyx}, \tag{4.19}$$

by using  $L_t^{-1}$  to both side into (4.19), we obtain that

$$u(x, y, t) = \sqrt{\frac{3\lambda}{2}} \sinh\left(\frac{x+y}{6}\right) + \int_0^t (A_n - 2B_n - 2C_n)dt. \tag{4.20}$$

From (4.20) we have

$$u_0 = \sqrt{\frac{3\lambda}{2}} \sinh\left(\frac{x+y}{6}\right), \tag{4.21}$$

and

$$u_{n+1} = \int_0^t (A_n - 4B_n) dt, \tag{4.22}$$

where,  $B_n = C_n$ .

By using (3.5) and (4.22), we calculate that

$$\left\{ \begin{array}{l} u_1 = -\frac{\lambda t}{6} \sqrt{\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right), \\ u_2 = \frac{\lambda^2 t^2}{2 \times 6^2} \sqrt{\frac{3\lambda}{2}} \sinh\left(\frac{x+y}{6}\right), \\ u_3 = -\frac{\lambda^3 t^3}{6^4} \sqrt{\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right), \\ u_4 = \frac{\lambda^4 t^4}{6^4 4!} \sqrt{\frac{3\lambda}{2}} \sinh\left(\frac{x+y}{6}\right), \\ u_5 = -\frac{\lambda^5 t^5}{6^5 5!} \sqrt{\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right). \\ \cdot \\ \cdot \\ \cdot \end{array} \right. \tag{4.23}$$

From (4.21) and (4.23), the solution of the (4.17) its given by

$$\begin{aligned} u(x, y, t) &= u_0(x, y, t) + u_1(x, y, t) + u_2(x, y, t) + u_3(x, y, t) + u_4(x, y, t) + u_5(x, y, t) + \dots \\ &= \sqrt{\frac{3\lambda}{2}} \sinh\left(\frac{x+y}{6}\right) \left(1 + \frac{\lambda^2 t^2}{6^2 2!} + \frac{\lambda^4 t^4}{6^4 4!} + \frac{\lambda^6 t^6}{6^6 6!} + \dots\right) \\ &\quad - \sqrt{\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right) \left(\frac{\lambda t}{6} + \frac{\lambda^3 t^3}{6^3 3!} + \frac{\lambda^5 t^5}{6^5 5!} + \frac{\lambda^7 t^7}{6^7 7!} + \dots\right), \\ &= \sqrt{\frac{3\lambda}{2}} \left(\sinh\left(\frac{x+y}{6}\right) \cosh\left(\frac{\lambda t}{6}\right) - \cosh\left(\frac{x+y}{6}\right) \sinh\left(\frac{\lambda t}{6}\right)\right), \\ &= \sqrt{\frac{3\lambda}{2}} \sinh\left(\frac{x+y-\lambda t}{6}\right). \end{aligned} \tag{4.24}$$

**Example 4.4** Solve the equation

$$u_t - (u^3)_x + 2(u^3)_{3x} + 2(u^3)_{yyx} = 0, \tag{4.25}$$

the initial condition:

$$u_0(x, y, t) = u(x, y, 0) = \sqrt{-\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right), \quad \lambda \text{ is constant.} \tag{4.26}$$

**Solution:**

$$L_t u = (u^3)_x - 2(u^3)_{3x} - 2(u^3)_{yyx}, \tag{4.27}$$



by using  $L_t^{-1}$  to both side into (4.27),we obtain that

$$u(x, y, t) = \sqrt{-\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right) + \int_0^t (A_n - 28B_n - 2C_n)dt. \quad (4.28)$$

From (4.28)we have

$$u_0 = \sqrt{-\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right), \quad (4.29)$$

and

$$u_{n+1} = \int_0^t (A_n - 4B_n)dt, \quad (4.30)$$

where,  $B_n = C_n$ .

By using (3.5) and (4.30), its easy to calculate that

$$\left\{ \begin{array}{l} u_1 = -\frac{\lambda t}{6} \sqrt{-\frac{3\lambda}{2}} \sinh\left(\frac{x+y}{6}\right), \\ u_2 = \frac{\lambda^2 t^2}{2 \times 6^2} \sqrt{-\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right), \\ u_3 = -\frac{\lambda^3 t^3}{6^4} \sqrt{-\frac{3\lambda}{2}} \sinh\left(\frac{x+y}{6}\right), \\ u_4 = \frac{\lambda^4 t^4}{6^4 4!} \sqrt{-\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right), \\ u_5 = -\frac{\lambda^5 t^5}{6^5 5!} \sqrt{-\frac{3\lambda}{2}} \sinh\left(\frac{x+y}{6}\right). \\ \cdot \\ \cdot \\ \cdot \end{array} \right. \quad (4.31)$$

From (4.29) and (4.31), the solution of the (4.25) its given by

$$\begin{aligned} u(x, y, t) &= u_0(x, y, t) + u_1(x, y, t) + u_2(x, y, t) + u_3(x, y, t) + u_4(x, y, t) + u_5(x, y, t) + \dots \\ &= \sqrt{-\frac{3\lambda}{2}} \cosh\left(\frac{x+y}{6}\right) \left(1 + \frac{\lambda^2 t^2}{6^2 2!} + \frac{\lambda^4 t^4}{6^4 4!} + \frac{\lambda^6 t^6}{6^6 6!} + \dots\right) \\ &\quad - \sqrt{-\frac{3\lambda}{2}} \sinh\left(\frac{x+y}{6}\right) \left(\frac{\lambda t}{6} + \frac{\lambda^3 t^3}{6^3 3!} + \frac{\lambda^5 t^5}{6^5 5!} + \frac{\lambda^7 t^7}{6^7 7!} + \dots\right), \\ &= \sqrt{-\frac{3\lambda}{2}} \left(\cosh\left(\frac{x+y}{6}\right) \cosh\left(\frac{\lambda t}{6}\right) - \sinh\left(\frac{x+y}{6}\right) \sinh\left(\frac{\lambda t}{6}\right)\right), \\ &= \sqrt{-\frac{3\lambda}{2}} \cosh\left(\frac{x+y-\lambda t}{6}\right). \end{aligned} \quad (4.32)$$

## 5 Discussion and Conclusion

The ZK-equation is introduced in 1974, and since that time there are many methods are used for solving this equation, these methods are of considerable advantages and some disadvantages, can

be summarized as:-

- 1- The difficulties arise in solving ZK-equation for higher order.
- 2- The computational operation of the methods is compacted.
- 3- Consumption of time in solving the problem.

The Adomian decomposition method is more effective than others for these reasons :-

- 1- It can be used for solving ZK-equation with higher orders.
- 2 The computational operations are simple.
- 3- The Adomian decomposition method express the solution in a series form, so we can predict the solution in a compact form.
- 4- Needs less time for solving problems.

The Adomian decomposition method considered as one of the strongest methods for solving ZK-equation, in spite it consists of more than one nonlinear term.

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## Competing Interests

Authors have declared that no competing interests exist.

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