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Asymptotical Wave Speed and Monotone Property of Traveling Wave Solution for a Two-Species Ratio-Dependent Predator-Prey System with Free Diffusion and Discrete Delay

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Abstract

In this paper, we consider a two-species ratio-dependent predator-prey system with free diffusion and discrete time delay. We study the asymptotical wave speed to give the necessary condition on the front speed, and prove that the traveling wave solution by combining the approach introduced by Canosa with the method of upper and lower solutions is monotone. Finally, we give a conclusion to summarize the achievements of the work.

Keywords: Asymptotical wave speed, delay, upper and lower solutions, traveling wave solution.

1 Introduction

In the natural world, there are many species whose individual members have a life history that takes them through two stages: immature and mature, such as some amphibious animals, which exhibit the above two stages. To investigate the above important phenomenon of species, some researchers introduce one delay or many delays to the Lotka-Volterra equations [1-5] to obtain delayed ordinary differential equations (DDEs, or called by retarded functional differential equations (RFDEs)). For the details, one can refer to [6-10] and so on.

Also, we remark that the specie's diffusion, which is that each specie's natural tendency is to move from the areas of bigger population concentration to ones of smaller population concentration, is an important phenomenon of species. So, following the authors of [11-17] to add diffusion terms, and considering the stage structure, we derive the following delayed reaction-diffusion equations:

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$$\begin{cases} \dfrac{\partial u_1}{\partial t} - D_0 \Delta u_1 = \alpha u_2(x,t) - \gamma u_1(x,t) - \alpha e^{-\gamma\tau} u_2(x,t-\tau), \\ \dfrac{\partial u_2}{\partial t} - D_1 \Delta u_2 = \alpha e^{-\gamma\tau} u_2(x,t-\tau) - \beta u_2^2(x,t) - \dfrac{c_0 u_2(x,t) v(x,t)}{u_2(x,t) + m v(x,t)}, \\ \dfrac{\partial v}{\partial t} - D_2 \Delta v = v(x,t)(-d + \dfrac{f u_2(x,t)}{u_2(x,t) + m v(x,t)}), \\ u_1(x,0) = \varphi_1(x) > 0, v(x,0) = \varphi_2(x) > 0, \\ u_2(x,t) = \varphi_3(x,t) \geq 0, -\tau \leq t \leq 0, \end{cases} \tag{1.1}$$

where $u_1(x,t), u_2(x,t)$ represent the densities of the immature and mature prey populations, respectively; $v(x,t)$ represents the density of predator population; $f > 0$, is the transformation coefficient of mature predator population; $\alpha e^{-\gamma\tau} u_2(t-\tau)$ represents the immatures who were born at time $t-\tau$ and survive at time t (with the immature death rate γ), and τ represents the transformation of immatures to matures; $\alpha > 0$, is the birth rate of the immature prey population; $\gamma > 0$,is the death rate of the immature prey population; $\beta > 0$ represents the mature death and overcrowding rate; the positive constants D_0, D_1 and D_2 are called diffusion coefficients, $d > 0, m > 0$ and $x \in$ □ . The initial data $\varphi_1(x), \varphi_2(x)$ and $\varphi_3(x,t)(-\tau \leq t \leq 0)$ are bounded and piecewise-continuous with a finite number of points of discontinuity.

By the way, such models or similar models involving delays and free diffusion are increasingly applied to a variety of situations, such as infectious disease dynamics, porous medium, chemical reaction, engineering control theory and others fields.

Note that $u_2(x,t)$ and $v(x,t)$ are independent of $u_1(x,t)$ but determine $u_1(x,t)$, hence, we can obtain the behavior of the solutions of the system (1.1) by studying the subsystem (1.2). Denote $u_2(x,t), v(x,t)$ by $u_1(x,t), u_2(x,t)$, respectively, and so is the initial data, then we get

$$\begin{cases} \dfrac{\partial u_1}{\partial t} - D_1 \Delta u_1 = \alpha e^{-\gamma\tau} u_1(x,t-\tau) - \beta u_1^2(x,t) - \dfrac{c_0 u_1(x,t) u_2(x,t)}{u_1(x,t) + m u_2(x,t)}, \\ \dfrac{\partial u_2}{\partial t} - D_2 \Delta u_2 = u_2(x,t)(-d + \dfrac{f u_1(x,t)}{u_1(x,t) + m u_2(x,t)}), \\ \\ u_2(x,0) = \varphi_2(x,0), u_1(x,t) = \varphi_1(x,t), x \in R, -\tau \le t \le 0. \end{cases} \tag{1.2}$$

Before proceeding further, let us nondimensionalize the system (1.2) with the scaling $U_1 = \beta u_1, U_2 = m\beta u_2, T = t,$ and denote U_1, U_2, T by u_1, u_2, t , respectively, we have

$$\begin{cases} \dfrac{\partial u_1}{\partial t} - D_1 \Delta u_1 = \alpha u_1(x,t-\tau) - u_1^2(x,t) - \dfrac{b u_1(x,t) u_2(x,t)}{u_1(x,t) + u_2(x,t)} \\ \dfrac{\partial u_2}{\partial t} - D_2 \Delta u_2 = u_2(x,t)(-d + \dfrac{f u_1(x,t)}{u_1(x,t) + u_2(x,t)}), \\ \\ u_2(x,0) = \varphi_2(x,0), u_1(x,t) = \varphi_1(x,t), x \in I, -\tau \le t \le 0, \end{cases} \tag{1.3}$$

where $a = \alpha e^{-\gamma\tau}, b = \dfrac{c_0}{m}.$

The existence of traveling wave solution of the system (1.3) is difficult and interesting problem [18]. Motivated by the results of [18], we study the existence of traveling wave solution of the two-species delayed system (1.3) . The key idea is to couple the uniformly approximated approach introduced by J. Canosa in [19] with the method of upper and lower solutions. The difficult issue is to construct the upper and lower solutions of the system (1.3) which has some suitable continuity.

The remaining parts of this paper are organized as follows. In Section 2, we prove that the traveling wave solution of the system (1.3) exist and appear to be monotone. Finally, we draw a conclusion summarizing the overall achievements of the work.

2 Traveling Wave Solution

2.1 Asymptotical Stability of Nonnegative Equilibria

Firstly, we discuss the asymptotical stability of the nonnegative equilibria by the linearized method. It is easy to check that the system (1.3) has an equilibrium $E_1(a,0)$ and a unique the positive equilibrium $E_2(c_1^*, c_2^*)$ if $f > d$ and $\frac{a}{b}+\frac{d}{f}>1$, where $c_1^*=\frac{(a-b)f+bd}{f}$, $c_2^*=\frac{(f-d)c_1^*}{d}$.

To use the linearized technique [7], we set $U(t)=(u_1(x,t),u_2(x,t)) \quad -E_i(i=1,2)$ and U_t=U(t+θ)(-$\tau \le \theta \le 0$), so we get the partial functional differential equation in $C \square C([-\tau,0];R^2)$ as follows

$$\frac{d}{dt}U(t)=D\Delta U(t)+N(\tau)(U_t)+f_0(U(t),\tau), \tag{2.1}$$

where $D=diag(D_1,D_2)$, $f_0: C\times R^+ \to R^2$ is a nonlinear operator,and $N(\tau): C\to R^2$ is a linear operator given by

$$N(\tau)(\varphi)=\begin{pmatrix} -\frac{b(c_1^*)^2}{(c_1^*+c_2^*)^2}\varphi_2(0)-\frac{b(c_2^*)^2}{(c_1^*+c_2^*)^2}\varphi_1(0)+a\varphi_1(-\tau)-2c_1^*\varphi_1(0) \\ \frac{f(c_2^*)^2}{(c_1^*+c_2^*)^2}\varphi_1(0)+\frac{f(c_1^*)^2}{(c_1^*+c_2^*)^2}\varphi_2(0)-d\varphi_2(0) \end{pmatrix} \tag{2.2}$$

for all $\varphi=(\varphi_1,\varphi_2)\in C$.

So, the characteristic equation for the linear equation $\dot{U}(t)=D\Delta U(t)+N(\tau)(U_t)$ is equivalent to

$$\left(\lambda+\mu_k D_1-ae^{-\lambda r}+2c_1^*+\frac{b(c_2^*)^2}{(c_1^*+c_2^*)^2}\right)\left(\lambda+\mu_k D_2+d-\frac{f(c_1^*)^2}{(c_1^*+c_2^*)^2}\right)+\frac{bf(c_1^*c_2^*)^2}{(c_1^*+c_2^*)^4}=0, \tag{2.3}$$

where $\mu_k(k=1,2,3,\cdots)$ is the eigenvalues of the operator $-\Delta$ on Ω with the homogeneous Neumann boundary condition such that $\mu_1<\mu_2<\mu_3<\cdots<\mu_n<\cdots$.

By determining the sign of λ of (2.3) at the equilibrium $E_i(i=1,2)$, omitting the detailed derivation (for the similar case, one can refer to[19], we have

Theorem2.1. If $af \geq \max\{2ad, 2b(f-d)\}$, , then the positive equilibrium $E_2(c_1^*, c_2^*)$ is locally asymptotically stable; if $f \geq d$,then the equilibrium $E_1(a,0)$ is unstable, moreover, if $f < d$,then the equilibrium $E_1(a,0)$ is locally asymptotically stable.

2.2 Asymptotical Wave Speed of Traveling Wave Solution

To seek a pair of traveling wave solution of the system (1.3) of the form $u_1(x,t) = \phi_1(s), u_2(x,t) = \phi_2(s),$ with $s = x + ct$ and c is the wave speed. So, we have

$$\begin{cases} D_1\phi_1''(s) - c\phi_1'(s) - \phi_1^2(s) - \dfrac{b\phi_1(s)\phi_2(s)}{\phi_1(s)+\phi_2(s)} + a\phi_1(s-c\tau) = 0, \\ D_2\phi_2''(s) - c\phi_2'(s) - d\phi_2(s) + \dfrac{f\phi_1(s)\phi_2(s)}{\phi_1(s)+\phi_2(s)} = 0, \\ \\ \phi_1(-\infty) = 0, \phi_1(+\infty) = c_1^*, \phi_2(-\infty) = 0, \phi_2(+\infty) = c_2^*. \end{cases} \tag{2.4}$$

For the system(2.4), we are interested in the minimum wave speed and it will decrease or increase when delay varies. Next, we give a necessary condition on the front speed c ahead of the front for the case of $s \to -\infty$. To do this, we seek the solutions of the proportional to $\begin{pmatrix} c_1^* \\ c_2^* \end{pmatrix} e^{\lambda s}$ to get

$$D_1\lambda^2 - c\lambda - c_1^* e^{\lambda s} - \frac{bc_2^*}{c_1^*+c_2^*} + ae^{-\lambda c\tau} = 0, \tag{2.5}$$

$$D_2\lambda^2 - c\lambda - d + \frac{fc_1^*}{c_1^*+c_2^*} = 0. \tag{2.6}$$

In order to have a front $\begin{pmatrix} \phi_1(s) \\ \phi_2(s) \end{pmatrix}$ that tends to 0 as $s \to -\infty$ without oscillating, it will be necessary for (2.5) and (2.6) to have some real positive roots. So, if τ is zero, then c and λ satisfy

$$D_1\lambda^2 - c\lambda - \frac{bc_2^*}{c_1^*+c_2^*} + a = 0 \tag{2.7}$$

Using (2.7)and (2.6), we have

$$c = D_2\sqrt{\frac{c_1^*}{D_2 - D_1}}, \lambda = \frac{D_2 c_1^*}{c(D_2 - D_1)}, \tag{2.8}$$

which needs $D_2 > D_1$.
For small τ, using(2.5) and(2.6),we get

$$(D_2 - D_1)\lambda^2 = c_1^* - a(1 - e^{-\lambda^2 D_2 \tau}) \approx c_1^* - a\lambda^2 D_2 \tau \tag{2.9}$$

Using(2.6)and (2.9), we have

$$\lambda = \sqrt{\frac{c_1^*}{aD_2\tau + D_2 - D_1}}, c = D_2\sqrt{\frac{c_1^*}{aD_2\tau + D_2 - D_1}}. \tag{2.10}$$

Thus, we see that the wave is slowed down by the small delay τ.

2.3 Monotone Traveling Wave Solution

In the subsection, we discuss the existence of monotone traveling wave solution by constructing a uniformly valid asymptotic approximation to the wavefronts (following the approach of [19]), which connect the zero solution with the positive steady state. Let $\eta = \sqrt{\varepsilon} s = \frac{s}{c}$ and seek a pair of solutions of the following form

$$\begin{pmatrix} \phi_1(s) \\ \phi_2(s) \end{pmatrix} = \begin{pmatrix} \psi_1(\eta) \\ \psi_2(\eta) \end{pmatrix}. \tag{2.11}$$

Substituting (2.11)into (2.4) yields

$$\begin{cases} \varepsilon D_1 \psi_1''(\eta) - \psi_1'(\eta) - \psi_1^2(\eta) - \dfrac{b\psi_1(\eta)\psi_2(\eta)}{\psi_1(\eta) + \psi_2(\eta)} + a\psi_1(\eta - \tau) = 0, \\ \varepsilon D_2 \psi_2''(\eta) - \psi_2'(\eta) - d\psi_2(\eta) + \dfrac{f\psi_1(\eta)\psi_2(\eta)}{\psi_1(\eta) + \psi_2(\eta)} = 0, \\ \psi_1(-\infty) = 0, \psi_1(+\infty) = c_1^*, \psi_2(-\infty) = 0, \psi_2(+\infty) = c_2^*. \end{cases} \tag{2.12}$$

Denote

$$\psi_1(\eta, \varepsilon) = \psi_{10}(\eta) + \varepsilon\psi_{11}(\eta) + \cdots, \psi_2(\eta, \varepsilon) = \psi_{20}(\eta) + \varepsilon\psi_{21}(\eta) + \cdots, \tag{2.13}$$

Substituting (2.13) into (2.12) by grouping the same order of ε, for the case ε^0 we have

$$\begin{cases} \psi_{10}'(\eta) = a\psi_{10}(\eta-\tau) - \psi_{10}^2(\eta) - \dfrac{b\psi_{10}(\eta)\psi_{20}(\eta)}{\psi_{10}(\eta)+\psi_{20}(\eta)}, \\ \psi_{20}'(\eta) = -d\psi_{20}(\eta) + \dfrac{f\psi_{10}(\eta)\psi_{20}(\eta)}{\psi_{10}(\eta)+\psi_{20}(\eta)}, \\ \\ \psi_{10}(-\infty)=0, \psi_{10}(+\infty)=c_1^*, \psi_{20}(-\infty)=0, \psi_{20}(+\infty)=c_2^*. \end{cases} \tag{2.14}$$

Remarks 2.1 For the case of large enough c, that is, $\varepsilon = \dfrac{1}{c^2}$ is a small parameter. So, the system (2.14) is uniformly valid asymptotic approximation to the system (2.12).

For convenience, denote $\psi_{10}(\eta), \psi_{20}(\eta)$ by $\psi_1(\eta), \psi_2(\eta)$, respectively, then we get

$$\begin{cases} \psi_1'(\eta) = \alpha e^{-\gamma\tau}\psi_1(\eta-\tau) - \psi_1^2(\eta) - \dfrac{b\psi_1(\eta)\psi_2(\eta)}{\psi_1(\eta)+\psi_2(\eta)}, \\ \psi_2'(\eta) = -d\psi_2(\eta) + \dfrac{f\psi_1(\eta)\psi_2(\eta)}{\psi_1(\eta)+\psi_2(\eta)}, \\ \\ \psi_1(-\infty)=0, \psi_1(+\infty)=c_1^*, \psi_2(-\infty)=0, \psi_2(+\infty)=c_2^*. \end{cases} \tag{2.15}$$

Theorem 2.2 If $af \ge \max\{2ad, 2b(f-d)\}$, then the positive solution of (2.15) is nondecreasing for $\eta \in (-\infty, +\infty)$.

Proof: Follow the method of [20] and [21], we show that a pair of upper and lower solutions $\left(\overline{\psi_1}(\eta), \overline{\psi_2}(\eta)\right)^T$ *and* $\left(\underline{\psi}_1(\eta), \underline{\psi}_2(\eta)\right)^T$ exists. To do that, we define the set

$$\Gamma = \left\{ \psi \in C(R, R^2) : \lim_{\eta\to-\infty} \psi = 0, \lim_{\eta\to+\infty} \psi = c^* = (c_1^*, c_2^*) \right\}.$$

where ψ is nondecreasing in $\mathbb{R}$, and $\psi = \begin{pmatrix} \psi_1(\eta) \\ \psi_2(\eta) \end{pmatrix}$.

Define

$$\overline{\psi}_1(\eta) = \begin{cases} \dfrac{c_1^*}{2} e^{\lambda\eta}, \eta \le 0, \\ c_1^* - \dfrac{c_1^*}{2} e^{-\lambda\eta}, \eta > 0, \end{cases} \qquad \overline{\psi}_2(\eta) = \begin{cases} \dfrac{c_2^*}{2} e^{\lambda\eta}, \eta \le 0 \\ c_2^* - \dfrac{c_2^*}{2} e^{-\lambda\eta}, \eta > 0, \end{cases} \tag{2.16}$$

Where

$$\lambda > \max\left\{\alpha e^{-\gamma\tau}, d, 2c_1^*\right\} \tag{2.17}$$

Using (2.16), we have

$$\overline{\psi}_1'(\eta) = \begin{cases} \frac{\lambda c_1^*}{2} e^{\lambda\eta}, \eta \le 0, \\ \frac{\lambda c_1^*}{2} e^{-\lambda\eta}, \eta > 0, \end{cases} \quad \overline{\psi}_2'(\eta) = \begin{cases} \frac{\lambda c_2^*}{2} e^{\lambda\eta}, \eta \le 0, \\ \frac{\lambda c_2^*}{2} e^{-\lambda\eta}, \eta > 0. \end{cases} \tag{2.18}$$

From (2.18), we know that $\overline{\psi} = \begin{pmatrix} \overline{\psi}_1(\eta) \\ \overline{\psi}_2(\eta) \end{pmatrix} \in \Gamma$. Next, we check that $\overline{\psi}$ is an upper solution of (2.15). Substituting (2.18) into the system(2.15) and using (2.17), for $\eta \le 0$ we have

$$\begin{aligned} &\overline{\psi}_1'(\eta) - \alpha e^{-\gamma\tau}\overline{\psi}_1(\eta-\tau) + \overline{\psi}_1^2(\eta) + \frac{b\overline{\psi}_1(\eta)\overline{\psi}_2(\eta)}{\overline{\psi}_1(\eta)+\overline{\psi}_2(\eta)} \\ &= \frac{\lambda c_1^*}{2} e^{\lambda\eta} - \frac{a c_1^*}{2} e^{\lambda\eta} e^{-\lambda\tau} + \left(\frac{c_1^*}{2} e^{\lambda\eta}\right)^2 + \frac{b c_1^* c_2^* e^{\lambda\eta}}{2(c_1^*+c_2^*)} \\ &\ge \frac{c_1^*}{2} e^{\lambda\eta}\left[\lambda - a + \frac{c_1^*}{2} e^{\lambda\eta} + \frac{b c_2^*}{c_1^*+c_2^*}\right] \\ &= \frac{c_1^*}{2} e^{\lambda\eta}\left[\lambda - a\left(1-\frac{e^{\lambda\eta}}{2}\right) + \frac{b c_2^*}{c_1^*+c_2^*}\left(1-\frac{e^{\lambda\eta}}{2}\right)\right] > 0, \end{aligned} \tag{2.19}$$

and

$$\begin{aligned} \overline{\psi}_2'(\eta) + d\overline{\psi}_2(\eta) - \frac{f\overline{\psi}_1(\eta)\overline{\psi}_2(\eta)}{\overline{\psi}_1(\eta)+\overline{\psi}_2(\eta)} &= \frac{c_2^*}{2} e^{\lambda\eta}\left[\lambda + d - \frac{f c_1^*}{c_1^*+c_2^*}\right] \\ &= \lambda c_2^* e^{\lambda\eta} > 0 \end{aligned} \tag{2.20}$$

For $\eta > \tau$, we obtain

$$
\begin{aligned}
&\bar{\psi}_1'(\eta) - a\bar{\psi}_1(\eta-\tau) + \bar{\psi}_1^2(\eta) + \frac{b\bar{\psi}_1(\eta)\bar{\psi}_2(\eta)}{\bar{\psi}_1(\eta)+\bar{\psi}_2(\eta)} \\
&\geq \frac{c_1^*}{2}e^{-\lambda\eta} + \frac{\alpha e^{-\gamma\tau}}{2}c_1^* e^{-\lambda(\eta-\tau)} - (c_1^*)^2 e^{-\lambda\eta} + (\frac{c_1^*}{2}e^{-\lambda\eta})^2 \\
&\quad - \frac{bc_1^*c_2^*}{c_1^*+c_2^*}e^{-\lambda\eta} + \frac{bc_1^*c_2^*}{c_1^*+c_2^*}(\frac{e^{-\lambda\eta}}{2})^2 \\
&\geq \frac{c_1^*}{2}e^{-\lambda\eta}\left[\lambda + \alpha e^{-\gamma\tau}e^{\lambda\tau} - 2c_1^* - \frac{2bc_2^*}{c_1^*+c_2^*}\right] \\
&= \frac{c_1^*}{2}e^{-\lambda\eta}\left[\lambda + \alpha e^{-\gamma\tau}e^{\lambda\tau} - 2\alpha e^{-\gamma\tau}\right] > 0.
\end{aligned} \tag{2.21}
$$

As for the case $0 < \eta \leq \tau$, using (2.17), we get

$$
\begin{aligned}
&\bar{\psi}_1'(\eta) - a\bar{\psi}_1(\eta-\tau) + \bar{\psi}_1^2(\eta) + \frac{b\bar{\psi}_1(\eta)\bar{\psi}_2(\eta)}{\bar{\psi}_1(\eta)+\bar{\psi}_2(\eta)} \\
&\geq \frac{c_1^*}{2}e^{-\lambda\eta} + \frac{\alpha e^{-\gamma r}}{2}c_1^* e^{-\lambda(\eta-r)} - (c_1^*)^2 e^{-\lambda\eta} + (\frac{c_1^*}{2}e^{-\lambda\eta})^2 \\
&\quad - \frac{bc_1^*c_2^*}{c_1^*+c_2^*}e^{-\lambda\eta} + \frac{bc_1^*c_2^*}{c_1^*+c_2^*}(\frac{e^{-\lambda\eta}}{2})^2 \\
&\geq \frac{c_1^*}{2}e^{-\lambda\eta}\left[\lambda + \alpha e^{-\gamma r}e^{\lambda r} - 2c_1^* - \frac{2bc_2^*}{c_1^*+c_2^*}\right] \\
&= \frac{c_1^*}{2}e^{-\lambda\eta}\left[\lambda + \alpha e^{-\gamma r}e^{\lambda r} - 2\alpha e^{-\gamma r}\right] \\
&> 0.
\end{aligned} \tag{2.22}
$$

during the calculation we use the fact $c_1^* + \frac{bc_2^*}{c_1^*+c_2^*} = a$ and the following inequality

$$
\frac{m_1 m_2}{m_1+m_2} > \frac{m_3 m_4}{m_3+m_4}, m_1 > m_3 > 0, m_2 > m_4 > 0. \tag{2.23}
$$

Similarly, using (2.23) and (2.17), we have

$$\bar{\psi}_2'(\eta) + d\bar{\psi}_2(\eta) - \frac{f\bar{\psi}_1(\eta)\bar{\psi}_2(\eta)}{\bar{\psi}_1(\eta)+\bar{\psi}_2(\eta)} \geq \frac{\lambda c_2^*}{2}e^{-\lambda\eta} + dc_2^* - \frac{dc_2^*}{2}e^{-\lambda\eta} - \frac{fc_1^*c_2^*}{c_1^*+c_2^*}$$
$$= \frac{\lambda c_2^*}{2}e^{-\lambda\eta} - \frac{dc_2^*}{2}e^{-\lambda\eta} = \frac{c_2^*}{2}e^{-\lambda\eta}(\lambda - d)$$
$$> 0. \tag{2.24}$$

From the above discussion, we have shown that $\overline{\psi}$ is a pair of upper solutions. Next, we show that $\underline{\psi} = \left(\underline{\psi}_1(\eta), \underline{\psi}_2(\eta)\right)^T$ is a pair of lower solutions. To this end, we define

$$\psi_1(\eta) = \begin{cases} \xi\varepsilon e^{\lambda_1\eta}, & \eta < 0, \\ \varepsilon - \xi\varepsilon e^{-\lambda_1\eta}, & \eta \geq 0 \end{cases} \quad and \quad \underline{\psi}_2(\eta) = 0, \tag{2.25}$$

where

$$0 < \varepsilon < \min\left\{c_1^*(<a), \xi a e^{-\lambda_1\tau} - \lambda_1\xi\right\}, 0 < \lambda_1 < ae^{-\lambda_1\tau}, \tag{2.26}$$

and ξ is small enough.

From the definition(2.25), we get

$$\psi_1'(\eta) = \begin{cases} \lambda_1\xi\varepsilon e^{\lambda_1\eta}, & if\,\eta < 0, \\ \lambda_1\xi\varepsilon e^{-\lambda_1\eta}, & if\,\eta \geq 0 \end{cases} \tag{2.27}$$

Using (2.26)and(2.27), for $0 \leq \eta < \tau$ we have

$$\underline{\psi}_1'(\eta) - a\underline{\psi}_1(\eta-\tau) + (\underline{\psi}_1(\eta))^2 + \frac{b\underline{\psi}_1(\eta)\underline{\psi}_2(\eta)}{\underline{\psi}_1(\eta)+\underline{\psi}_2(\eta)}$$
$$= \lambda_1\xi\varepsilon e^{-\lambda_1\eta} - \alpha e^{-\gamma\tau}\xi\varepsilon e^{\lambda_1(\eta-\tau)} + (\varepsilon - \xi\varepsilon e^{-\lambda_1\eta})^2 \tag{2.28}$$
$$\leq \varepsilon(\lambda_1\xi - a\xi e^{-\lambda_1\tau} + \varepsilon)$$
$$< 0$$

And for $\eta \geq \tau$, we have

$$\begin{aligned}&\underline{\psi}_1'(\eta)-\alpha e^{-\gamma\tau}\underline{\psi}_1(\eta-\tau)+\left(\underline{\psi}_1(\eta)\right)^2+\frac{b\underline{\psi}_1(\eta)\underline{\psi}_2(\eta)}{\underline{\psi}_1(\eta)+\underline{\psi}_2(\eta)}\\&=\lambda_1\xi\varepsilon e^{-\lambda_1\eta}-\alpha e^{-\gamma\tau}\left(\varepsilon-\xi\varepsilon e^{-\lambda_1(\eta-\tau)}\right)+\left(\varepsilon-\xi\varepsilon e^{-\lambda_1\eta}\right)^2\\&\le 0.\end{aligned} \tag{2.29}$$

For $\eta<0$, we get

$$\begin{aligned}&\underline{\psi}_1'(\eta)-\alpha e^{-\gamma\tau}\underline{\psi}_1(\eta-\tau)+(\underline{\psi}_1(\eta))^2+\frac{b\underline{\psi}_1(\eta)\underline{\psi}_2(\eta)}{\underline{\psi}_1(\eta)+\underline{\psi}_2(\eta)}\\&=\lambda_1\xi\varepsilon e^{\lambda_1\eta}-\alpha e^{-\gamma\tau}\xi\varepsilon e^{\lambda_1(\eta-\tau)}+(\xi\varepsilon e^{\lambda_1\eta})^2\\&\le\xi\varepsilon e^{\lambda_1\eta}\left[\lambda_1-\alpha e^{-\lambda_1\tau}+\xi\varepsilon\right]\\&<0.\end{aligned} \tag{2.30}$$

From(2.26), we know that $\varepsilon<c_1^*$, which implies that $\underline{\psi}\le\overline{\psi}$ holds.

Next, we check that the right term $f(\psi)=\left(f_1(\psi),f_2(\psi)\right)^T$ is qusi-monotone, where

$$\begin{cases}f_1(\psi)=a\psi_1(-\tau)-\psi_1^2(0)-\dfrac{b\psi_1(0)\psi_2(0)}{\psi_1(0)+\psi_2(0)},\\[2ex] f_2(\psi)=-d\psi_2(0)+\dfrac{f\psi_1(0)\psi_2(0)}{\psi_1(0)+\psi_2(0)}.\end{cases} \tag{2.31}$$

For $\phi=(\phi_1,\phi_2)^T$ and $\psi=(\psi_1,\psi_2)^T\in C\left([-\tau,0],\mathbb{R}^2\right)$,

if $0\le\psi(\theta)\le\phi(\theta)\le c^*=\left(c_1^*,c_2^*\right)$ for $\theta\in[-\tau,0]$ and there exists a positive constant δ' such that $\phi_2(0)-\psi_2(0)<\delta'\left(\phi_1(0)-\psi_1(0)\right)$, then we have

$$f_2(\phi)-f_2(\psi)=-d\phi_2(0)+\frac{f\phi_1(0)\phi_2(0)}{\phi_1(0)+\phi_2(0)}+d\psi_2(0)-\frac{f\psi_1(0)\psi_2(0)}{\psi_1(0)+\psi_2(0)}$$
$$=-d(\phi_2(0)-\psi_2(0))+\frac{f\phi_1(0)\psi_1(0)}{(\phi_1(0)+\phi_2(0))(\psi_1(0)+\psi_2(0))}(\phi_2(0)-\psi_2(0))$$
$$+\frac{f\phi_2(0)\psi_2(0)}{(\phi_1(0)+\phi_2(0))(\psi_1(0)+\psi_2(0))}(\phi_1(0)-\psi_1(0))\geq -d(\phi_2(0)-\psi_2(0)). \qquad (2.32)$$

Taking $\delta_2 \geq d$ and using (2.32), we have

$$f_2(\phi)-f_2(\psi)+\delta_2(\phi_2^2(0)-\psi_2(0))$$
$$\geq(\delta_2-d)(\phi_2^2(0)-\psi_2(0))$$
$$\geq 0. \qquad (2.33)$$

Similarly, taking $\delta_1 \geq 2c_1^* + b + b\delta^{'}$ and using (2.31), we obtain

$$f_1(\phi)-f_1(\psi)+\delta_1(\phi_1(0)-\psi_1(0))$$
$$\geq(\delta_1-2c_1^*-b-b\delta')(\phi_1(0)-\psi_1(0))$$
$$\geq 0. \qquad (2.34)$$

Taking $\delta=\left(\delta_1,\delta_2\right)^T$, we have

$$f_c(\phi)-f_c(\psi)+\delta(\phi(0)-\varphi(0))\geq(\delta I-B)(\phi(0)-\varphi(0))\geq 0, \qquad (2.35)$$

where I is a 2×2 identity matrix and $B=diag(2c_1^*+b+b\delta', d)$..

From [20], we know that there exists at least one solution in the set Γ. The proof of the theorem is completed.

3 Conclusion

S.A. Gourley and Y. Kuang pointed out that the existence of wavefront solutions for this single specie model in[18] is an open question. In the paper, we considered the asymptotical behavior of traveling wave solution of a two-species delayed predator-prey system for the case of large enough wave speed c.

The free diffusion has no effect on the monotone property of traveling wave solution of a two-species delayed predator-prey system when the wave speed is large enough. However, from Subsection 2.2 we know that the sign of $D_2 - D_1$ and the delay τ affect the wave speed, which is an interesting problem.

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Competing Interests

Authors have declared that no competing interests exist.

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