

On the Rate of Convergence of Some New Modified Iterative Schemes

Renu Chugh, Sanjay Kumar*

Department of Mathematics, M.D.U, Rohtak, India
 Email: *sanjay.sanju84@gmail.com

Received September 2, 2013; revised October 2, 2013; accepted October 9, 2013

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ABSTRACT

In this article, following Bizare and Amriteimoori [1] and B. Parsad and R. Sahni [2], we modify Ishikawa, Agarwal *et al.*, Noor, SP iterative schemes and compare the rate of convergence of Ishikawa, Agarwal *et al.*, Noor, SP and new modified Ishikawa, Agarwal *et al.*, Noor, SP iterative schemes not only for particular fixed value of $\alpha_n, \beta_n, \gamma_n$ but also for varying the value of $\alpha_n, \beta_n, \gamma_n$. With the help of two numerical examples, we compare the converging step.

Keywords: Metric Space; New Modified Ishikawa; New Modified Agarwal *et al.*; New Modified SP; New Modified Noor

1. Introduction

Let X be a complete metric space and T be self map, then $F_T = \{x \in X, Tx = x\}$ is called set of fixed points of T . Now in literature, there are several iteration processes to find the fixed point of any equation. In complete metric space, Picard iteration process is defined as

$$x_{n+1} = T(x_n), n = 0, 1, \dots$$

which is used to approximate fixed points of mappings satisfying the condition

$$d(Tx, Ty) \leq ad(x, y), \text{ where } 0 \leq a < 1$$

called Banach contraction condition.

In Ishikawa iteration process [3], $\{x_n\}_{n=0}^{\infty}$ is defined as

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n T y_n \\ y_n &= (1 - \beta_n)x_n + \beta_n T x_n \end{aligned}$$

where α_n, β_n are real sequences in $[0, 1]$.

Now for Agarwal *et al.* iteration [4], $\{x_n\}_{n=0}^{\infty}$ is defined as

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)T x_n + \alpha_n T y_n \\ y_n &= (1 - \beta_n)x_n + \beta_n T x_n \end{aligned}$$

where α_n, β_n are real sequences in $[0, 1]$.

M. A. Noor defines [5] $\{x_n\}_{n=0}^{\infty}$ as

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n T y_n \\ y_n &= (1 - \beta_n)x_n + \beta_n T z_n \\ z_n &= (1 - \gamma_n)x_n + \gamma_n T x_n \end{aligned}$$

where $\alpha_n, \beta_n, \gamma_n$ are real sequences in $[0, 1]$.

For SP iteration [6] $\{x_n\}_{n=0}^{\infty}$ is defined as

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)y_n + \alpha_n T y_n \\ y_n &= (1 - \beta_n)z_n + \beta_n T z_n \\ z_n &= (1 - \gamma_n)x_n + \gamma_n T x_n \end{aligned}$$

where $\alpha_n, \beta_n, \gamma_n$ are real sequences in $[0, 1]$.

2. Preliminaries

In this paper following Bizare and Amriteimoori [1] and B. Parsad and R. Sahni [2] we prove the basic results in sequel. In [1] Bizare and Amriteimoori improved the picard iteration under following conditions:

- 1) Initial approximation is chosen in the interval $[a, b]$, where function is defined.
- 2) Function has continuous derivative on (a, b) .
- 3) $|T'(x)| < 1$ for all $x \in [a, b]$
- 4) $a \leq T(x) \leq b$ for all $x \in [a, b]$

*Corresponding author.

Definition 2.1 [1]. Let $\{x_n\}$ converges to α . If there exists an integer constant q and a real +ve constant C such that

$$\lim_{n \rightarrow \infty} \left| \frac{x_{n+1} - \alpha}{(x_n - \alpha)^q} \right| = C$$

q is called order and C is called constant of convergence

Theorem 2.2([1,7]). Let $f \in C^q[a, b]$, if $f^k(x) = 0$ for $k = 1, 2, \dots, q-1$ and $f^q(x) \neq 0$ then sequence $\{x_n\}$ is of order q .

To improve the order of convergence of fixed iterative schemes, such that

$f'(\alpha), f''(\alpha), \dots, f^k(\alpha) = 0$. We determines $\lambda_i (i = 1, 2, \dots, k)$ from the following equation

$$\begin{aligned} &x + \lambda_1 x + \lambda_2 x^2 + \dots + \lambda_k x^k \\ &= f(x) + \lambda_1 x + \lambda_2 x^2 + \dots + \lambda_k x^k \end{aligned}$$

which becomes

$$x = \frac{f(x) + \lambda_1 x + \lambda_2 x^2 + \dots + \lambda_k x^k}{1 + \lambda_1 + \lambda_2 x + \dots + \lambda_k x^{k-1}} = f_\lambda(x) \text{ this is fixed}$$

point equation form .Now the assumption that $f'_\lambda(\alpha) = f''_\lambda(\alpha) = \dots = f^{k-1}_\lambda(\alpha) = 0$ yields to a system of linear equations which after solving [1] converted into upper triangular matrix which have nonzero diagonal entries. It means determinant is nonzero. So we determine $\lambda_i (i = 1, 2, \dots, k)$ uniquely.

Now the new Picard iteration becomes

$$x_{n+1} = f_\lambda(x_n) \quad n = 1, 2, \dots \tag{1}$$

where

$$f_\lambda(x) = \frac{f(x) + \lambda_1 x + \lambda_2 x^2 + \dots + \lambda_k x^k}{1 + \lambda_1 + \lambda_2 x + \dots + \lambda_k x^{k-1}} \tag{2}$$

Following Bhagwati Parsad and Ritu Shani [2] the new modified Ishikawa, Agarwal *et al.*, Noor, SP iterations are defined as:

New modified Ishikawa iteration scheme

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n f_\lambda(y_n) \\ y_n &= (1 - \beta_n)x_n + \beta_n f_\lambda(x_n) \end{aligned} \tag{3}$$

New modified Agarwal *et al.* iteration scheme

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)f_\lambda(x_n) + \alpha_n f_\lambda(y_n) \\ y_n &= (1 - \beta_n)x_n + \beta_n f_\lambda(x_n) \end{aligned} \tag{4}$$

New modified Noor iteration scheme

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)x_n + \alpha_n f_\lambda(y_n) \\ y_n &= (1 - \beta_n)x_n + \beta_n f_\lambda(z_n) \\ z_n &= (1 - \gamma_n)x_n + \gamma_n f_\lambda(x_n) \end{aligned} \tag{5}$$

New modified SP iteration scheme

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n)y_n + \alpha_n f_\lambda(y_n) \\ y_n &= (1 - \beta_n)z_n + \beta_n f_\lambda(z_n) \\ z_n &= (1 - \gamma_n)x_n + \gamma_n f_\lambda(x_n) \end{aligned} \tag{6}$$

where $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\}$ are real sequences in $[0, 1]$.

In this article, we compare the rate of convergence of new modified iterative schemes and simple iterative schemes with the help of the following examples

$$p_1(x) = e^{(1-x)^2} - 1 - x \tag{7}$$

$$p_2(x) = \frac{3}{8} + \frac{35}{8}x^4 - \frac{15}{4}x^2 \tag{8}$$

To find the fixed point we write $p_1(x)$ and $p_2(x)$ as

$$e^{(1-x)^2} - 1 - x = 0 \quad \text{and} \quad \frac{3}{8} + \frac{35}{8}x^4 - \frac{15}{4}x^2 = 0$$

both equations has unique root in the interval $(0,1)$ so we convert this in the fixed point form

$$x = e^{(1-x)^2} - 1 = f(x) \quad \text{and} \quad x = \frac{\sqrt{90+1050x^4}}{30} = f(x)$$

and take $\alpha = 0.5$ and $\alpha = 0.35$ respectively. Now we solve it by

$$f_\lambda(x) = \frac{f(x) + \lambda_1 x + \lambda_2 x^2 + \dots + \lambda_k x^k}{1 + \lambda_1 + \lambda_2 x + \dots + \lambda_k x^{k-1}}$$

For respective value of $\alpha, \lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k$ can be determined uniquely from system of linear equations as in [5] for $\alpha = 0.5$ and $f(x) = e^{(1-x)^2} - 1$ we have

$$\begin{pmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 1 & 2\alpha & 3\alpha^2 & 4\alpha^3 \\ 0 & 0 & 2 & 6\alpha & 12\alpha^2 \\ 0 & 0 & 0 & 6 & 24\alpha \\ 0 & 0 & 0 & 0 & 24 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{pmatrix} = \begin{pmatrix} f'_\lambda(\alpha) \\ f^{(2)}_\lambda(\alpha) \\ f^{(3)}_\lambda(\alpha) \\ f^{(4)}_\lambda(\alpha) \\ f^{(5)}_\lambda(\alpha) \end{pmatrix} = \begin{pmatrix} 1.28403 \\ -3.85208 \\ 8.98818 \\ -32.1006 \\ 104.006 \end{pmatrix} \tag{9}$$

After solving the system of linear equations we have

$$\lambda_1 = 6.0858, \lambda_2 = -17757, \lambda_3 = 22.2698, \lambda_4 = -14.0173, \lambda_5 = 4.3336$$

for second polynomial equation where $\alpha = 0.35$ and

$$f(x) = \frac{\sqrt{90+1050x^4}}{30}$$

System of linear equations become

$$\begin{pmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 1 & 2\alpha & 3\alpha^2 & 4\alpha^3 \\ 0 & 0 & 2 & 6\alpha & 12\alpha^2 \\ 0 & 0 & 0 & 6 & 24\alpha \\ 0 & 0 & 0 & 0 & 24 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \end{pmatrix} = \begin{pmatrix} -f'_\lambda(\alpha) \\ -f_\lambda^{(2)}(\alpha) \\ -f_\lambda^{(3)}(\alpha) \\ -f_\lambda^{(4)}(\alpha) \\ -f_\lambda^{(5)}(\alpha) \end{pmatrix} = \begin{pmatrix} -0.291842 \\ -2.25304 \\ -8.53984 \\ 32.6662 \\ 422.235 \end{pmatrix} \tag{10}$$

$$\lambda_1 = 0.0129005, \lambda_2 = -0.330016, \lambda_3 = 3.01516, \lambda_4 = 17.5910, \lambda_5 = -19.186$$

3. Experiments

Now using the value of $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_k$ and iterative schemes we have following **Tables 1-24**, and **Figures 1-16**.

We take $\alpha_n = a, \beta_n = b, \gamma_n = c$

Hence we get

Table 1. Simple Ishikawa for $p_1(x)$.

N	a = 0.2, b = 0.9999		a = 0.2, b = 0.9		a = 0.3, b = 0.6		a = 0.2, b = 0.2	
	x_{n+1}	x_{n+1}	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.0804248	0.0100502	0.162309	0.0100502	0.446501	0.0100502	0.736069	0.0100502
1	0.56607	1.32942	0.519967	1.01723	0.41845	0.358473	0.622944	0.0721434
2	0.291442	0.207189	0.346672	0.259144	0.413572	0.402422	0.548051	0.152774
7	0.443316	0.483717	0.417293	0.426179	0.412392	0.412388	0.427266	0.375276
8	0.388159	0.363285	0.409449	0.404313	0.412391	0.412391	0.421969	0.388222
9	0.431032	0.454046	0.414152	0.417296	0.412391	0.412391	0.418561	0.39671
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27	0.412589	0.412814	0.412391	0.412392			0.412393	0.412385
28	0.412238	0.412063	0.412391	0.412391			0.412393	0.412387
29	0.41251	0.412646	0.412391	0.412391			0.412392	0.412389
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31	0.412463	0.412545					0.412392	0.41239
32	0.412335	0.412272					0.412391	0.412391
33	0.412435	0.412484					0.412391	0.412391
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55	0.412391	0.412392						
56	0.412391	0.412391						
57	0.412391	0.412391						

Table 2. Modified Ishikawa for $p_1(x)$.

N	$a = 0.1 \text{ to } 0.9, b = 0.99$		$a = 0.1 \text{ to } 0.9, b = 0.9$		$a = 0.1 \text{ to } 0.9, b = 0.6$		$a = 0.1 \text{ to } 0.9, b = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.424618	0.472629	0.481638	0.472629	0.654425	0.472629	0.942404	0.472629
1	0.412376	0.412344	0.416257	0.408094	0.488042	0.370271	0.886798	0.420223
2	0.412391	0.412391	0.412795	0.412411	0.440202	0.407234	0.834615	0.384598
3	0.412391	0.412391	0.412434	0.412395	0.423255	0.411826	0.786785	0.364432
4	0.412391	0.412391	0.412396	0.412392	0.41673	0.412365	0.743706	0.356248
5			0.412392	0.412391	0.414139	0.412411	0.705366	0.356001
6			0.412391	0.412391	0.413097	0.412404	0.671492	0.360233
7			0.412391	0.412391	0.412677	0.412397	0.641687	0.366495
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14					0.412392	0.412391	0.512968	0.400961
15					0.412391	0.412391	0.502184	0.403247
16					0.412391	0.412391	0.492625	0.405092
100							0.412403	0.412391
134							0.412392	0.412391
135							0.412391	0.412391
136							0.412391	0.412391

Table 3. Simple Agarwal *et al.* for $p_1(x)$.

N	$a = 0.2, b = 0.9999$		$a = 0.2, b = 0.9$		$a = 0.3, b = 0.6$		$a = 0.5, b = 0.5$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.0803358	0.0100502	0.0733136	0.0100502	0.090521	0.0100502	0.177931	0.0100502
1	0.566185	1.3298	0.644644	1.3602	0.727194	1.2868	0.583438	0.965599
2	0.291318	0.20707	0.222863	0.134597	0.177672	0.0772623	0.323279	0.189489
25	0.412721	0.413097	0.418074	0.423399	0.420618	0.428339	0.412391	0.412392
26	0.412135	0.411843	0.407474	0.403035	0.405294	0.398897	0.412391	0.412391
27	0.412591	0.412817	0.416645	0.42061	0.418546	0.424292	0.412391	0.412391
55	0.412391	0.412392	0.412465	0.412533	0.412498	0.412596		
56	0.412391	0.412391	0.412327	0.412269	0.412299	0.412214		
95			0.412391	0.412392	0.412391	0.412392		
96			0.412391	0.412391	0.412391	0.412391		
97					0.412391	0.412392		
98					0.412391	0.412391		

Table 4. Modified Agarwal *et al.* for $p_1(x)$.

N	$a = 0.9, b = 0.99$		$a = 0.1, b = 0.9$		$a = 0.3, b = 0.6$		$a = 0.9, b = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.408091	0.472629	0.428901	0.472629	0.408937	0.472629	0.465458	0.472629
1	0.412381	0.412331	0.412289	0.412252	0.412356	0.412346	0.410188	0.409941
2	0.412391	0.412391	0.41239	0.41239	0.412391	0.412391	0.412368	0.412365
3	0.412391	0.412391	0.412391	0.412391	0.412391	0.412391	0.412391	0.412391
4	0.412391	0.412391	0.412391	0.412391	0.412391	0.412391	0.412391	0.412391

Table 5. Simple SP for $p_1(x)$.

N	$a = 0.1, b = 0.1, c = 0.999$		$a = 0.1, b = 0.1, c = 0.9$		$a = 0.5, b = 0.5, c = 0.5$		$a = 0.1, b = 0.1, c = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.0736477	0.0100502	0.139578	0.0100502	0.416514	0.0100502	0.667545	0.0100502
1	0.701496	1.35874	0.616499	1.09662	0.412245	0.405588	0.531266	0.116866
2	0.182726	0.0931953	0.271814	0.158438	0.412396	0.412634	0.463428	0.245717
3	0.617407	0.950211	0.519974	0.699365	0.412391	0.412382	0.433328	0.333637
4	0.242048	0.157633	0.334615	0.259135	0.412391	0.412391	0.420799	0.378667
5	0.566158	0.776228	0.471698	0.556962	0.412391	0.412391	0.415737	0.398604
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16	0.371841	0.341091	0.409891	0.406887			0.412391	0.412391
17	0.448911	0.483766	0.41427	0.416556			0.412391	0.412391
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50	0.41149	0.41072	0.412391	0.412391				
51	0.413197	0.413889	0.412391	0.412391				
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126	0.412391	0.412391						

Table 6. Modified SP for $p_1(x)$.

N	$a = 0.9, b = 0.9, c = 0.9999$		$a = 0.9, b = 0.9, c = 0.9$		$a = 0.2, b = 0.9, c = 0.1$		$a = 0.1, b = 0.1, c = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.412399	0.472629	0.412496	0.472629	0.436509	0.472629	0.844335	0.472629
1	0.412391	0.412391	0.412391	0.412392	0.413946	0.411994	0.712973	0.367256
2	0.412391	0.412391	0.412391	0.412391	0.412512	0.412403	0.618933	0.359118
3	0.412391	0.412391	0.412391	0.412391	0.412401	0.412392	0.55555	0.37889
4	0.412391	0.412391	0.412391	0.412391	0.412392	0.412391	0.512865	0.394474
5					0.412391	0.412391	0.483669	0.403265
6					0.412391	0.412391	0.463373	0.407829
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43							0.412392	0.412391
44							0.412392	0.412391
45							0.412391	0.412391
46							0.412391	0.412391

Table 7. Simple Noor for $p_1(x)$.

N	$a = 0.3, b = 0.3, c = 0.5$		$a = 0.3, b = 0.5, c = 0.7$		$a = 0.5, b = 0.7, c = 0.9$		$a = 0.5, b = 0.7, c = 0.9$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.506337	0.0100502	0.449615	0.0100502	0.331926	0.0100502	0.381042	0.0100502
1	0.417997	0.275966	0.410239	0.353812	0.506867	0.562555	0.442088	0.466838
2	0.412427	0.40316	0.41258	0.415975	0.34732	0.275298	0.389512	0.365152
3	0.412391	0.412331	0.412375	0.412077	0.485348	0.531107	0.433466	0.451642
5	0.412391	0.412391	0.412391	0.412389	0.470593	0.508772	0.395663	0.378452
6	0.412391	0.412391	0.412391	0.412391	0.367544	0.323489	0.427488	0.440836
7			0.412391	0.412391	0.459703	0.491825	0.400159	0.38787
75							0.412391	0.412392
76							0.412391	0.412391
155					0.412391	0.412392		
156					0.412391	0.412391		
157					0.412391	0.412391		

Table 8. Modified Noor for $p_1(x)$.

N	$a = 0.9, b = 0.9, c = 0.999$		$a = 0.9, b = 0.9, c = 0.9$		$a = 0.7, b = 0.5, c = 0.5$		$a = 0.1, b = 0.1, c = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.410381	0.472629	0.469342	0.472629	0.681606	0.472629	0.942022	0.472629
1	0.412389	0.412368	0.418102	0.409542	0.539107	0.364431	0.886261	0.419927
2	0.412391	0.412391	0.412967	0.412409	0.473981	0.398109	0.834065	0.384326
3	0.412391	0.412391	0.412449	0.412396	0.442827	0.409028	0.78628	0.364285
6			0.412391	0.412391	0.416194	0.4124	0.67112	0.360296
7			0.412391	0.412391	0.4143	0.412411	0.641343	0.366574
20					0.412391	0.412391	0.463812	0.409495
21					0.412391	0.412391		
134							0.412392	0.412391
135							0.412391	0.412391

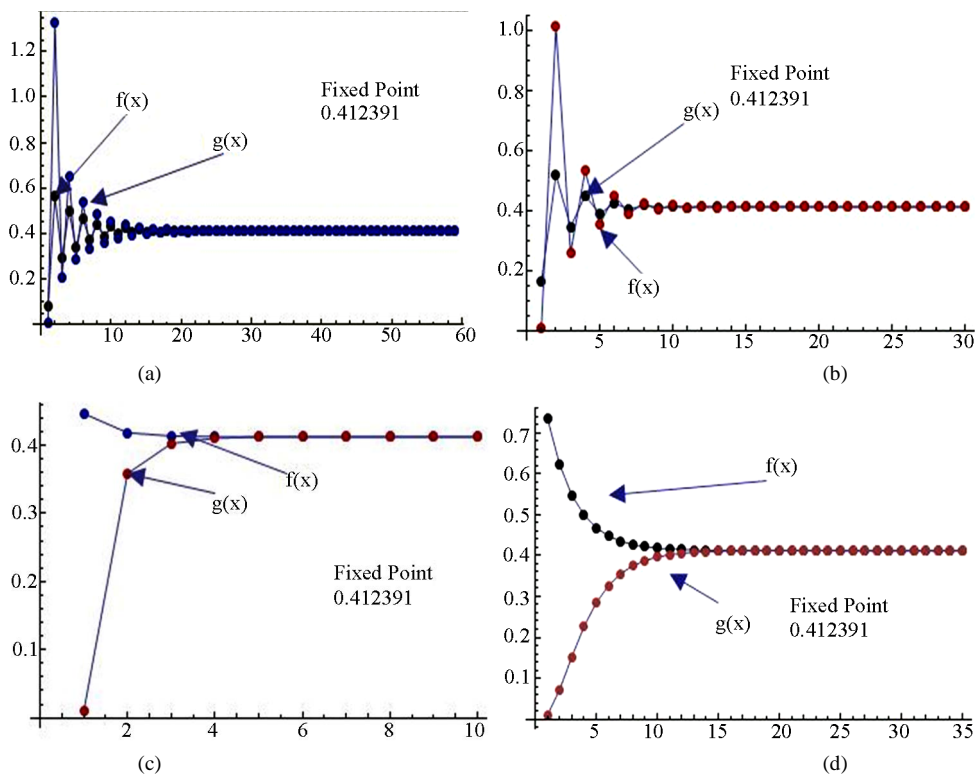


Figure 1. Graphical observations of simple Ishikawa iteration for $p_1(x)$. Here (a)-(d) show the graph for Table 1. The merging point with value 0.412391 is fixed point.

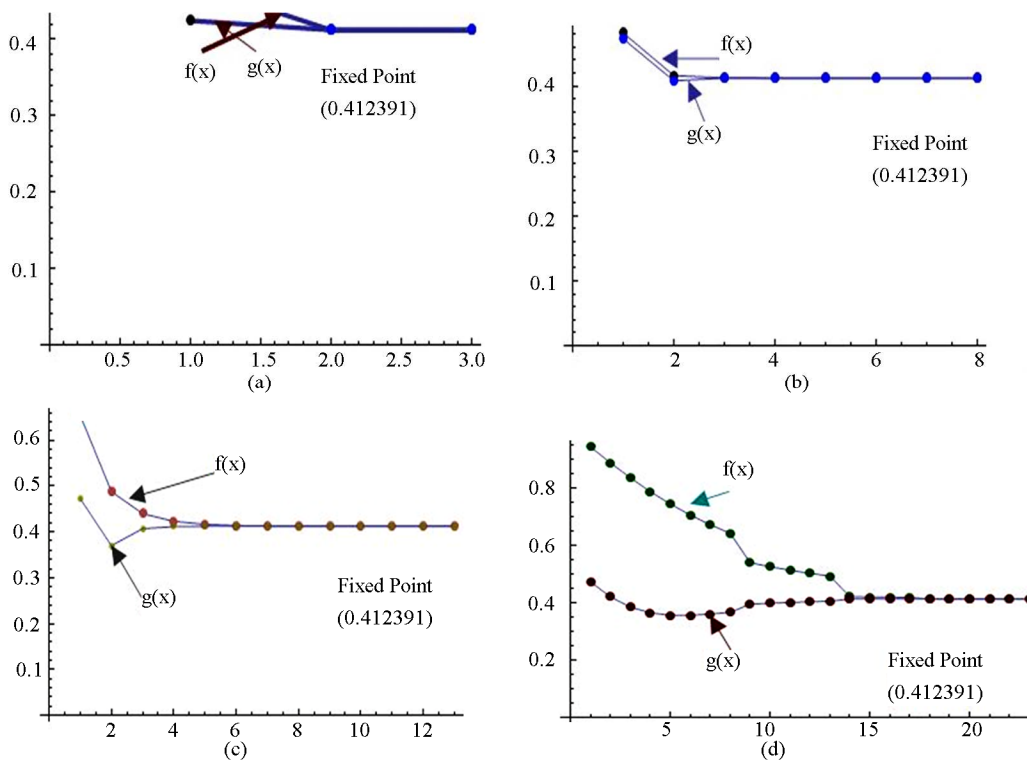


Figure 2. Graphical observations of new modified Ishikawa iteration for $p_1(x)$. Here (a)-(d) show the graph for Table 2. The merging point with value 0.412391 is fixed point.

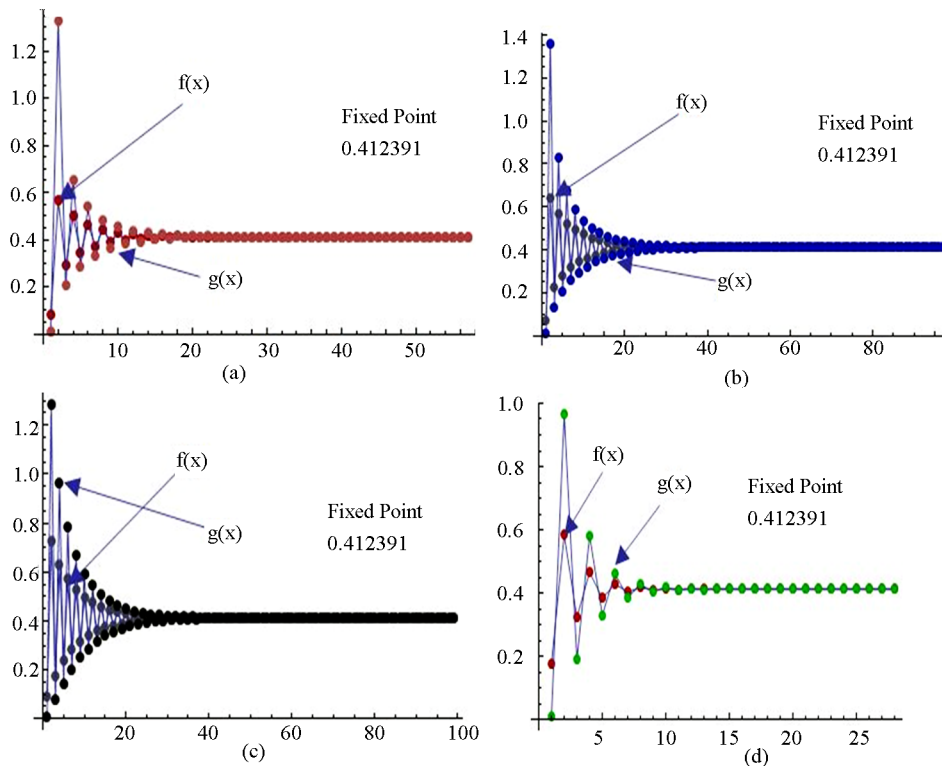


Figure 3. Graphical observations of simple Agarwal *et al.* iteration for $p_1(x)$. Here (a)-(d) show the graph for Table 3. The merging point with value 0.412391 is fixed point.

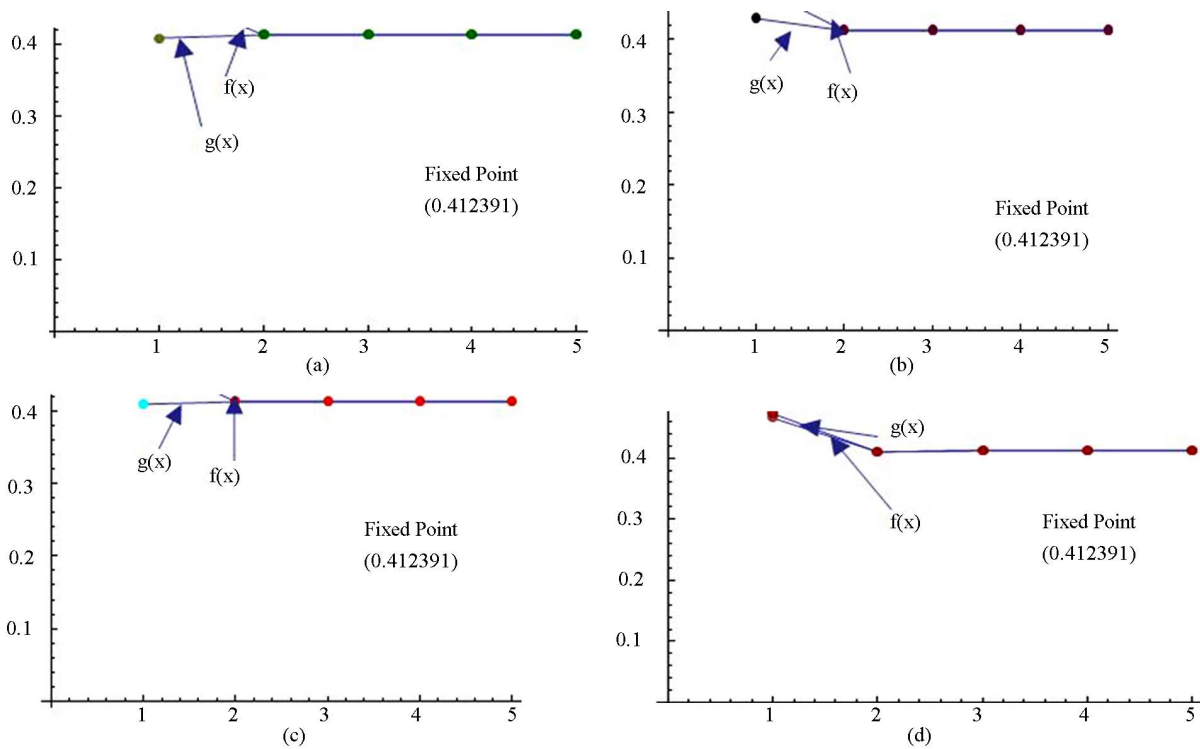


Figure 4. Graphical observations of new modified Agarwal *et al.* iteration for $p_1(x)$. Here (a)-(d) show the graph for Table 4. The merging point with value 0.412391 is fixed point.

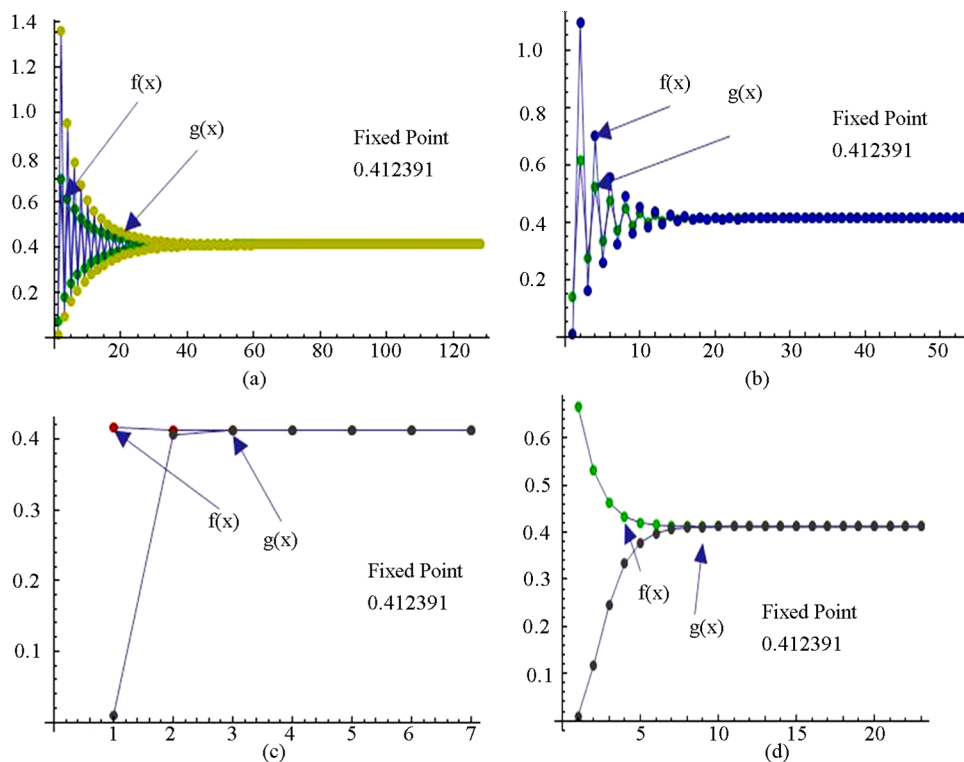


Figure 5. Graphical observations of simple SP iteration for $p_1(x)$. Here Here (a)-(d) show the graph for Table 5. The merging point with value 0.412391 is fixed point.

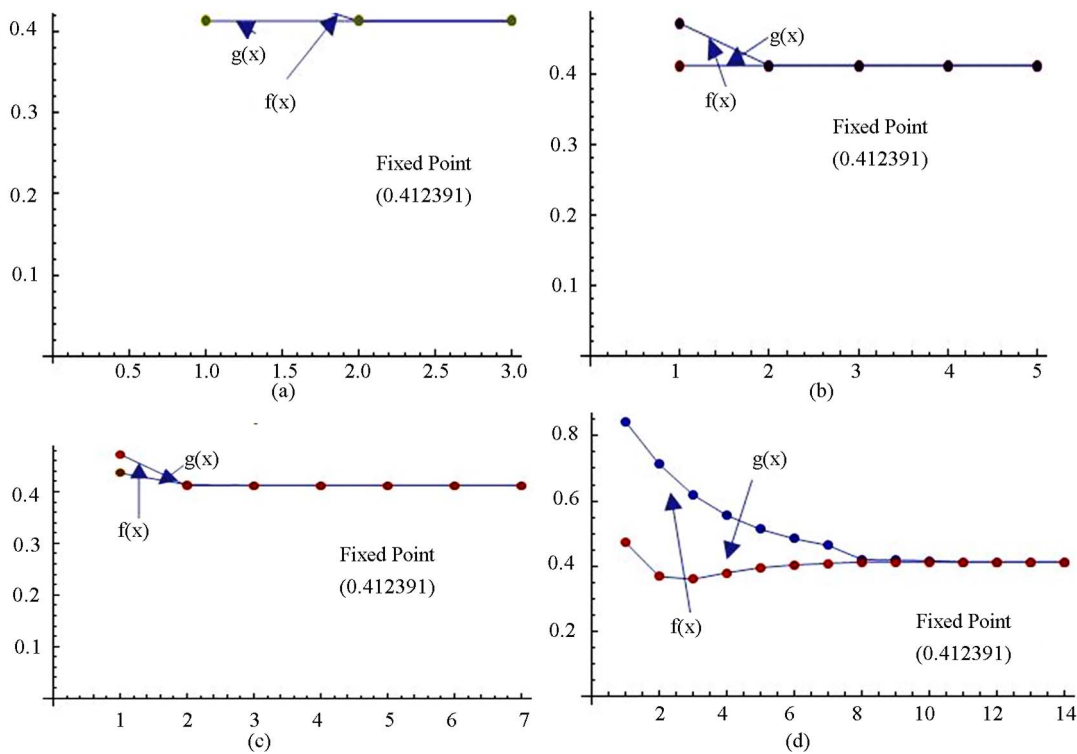


Figure 6. Graphical observations of new modified SP iteration for $p_1(x)$. Here (a)-(d) show the graph for Table 6. The merging point with value 0.412391 is fixed point.

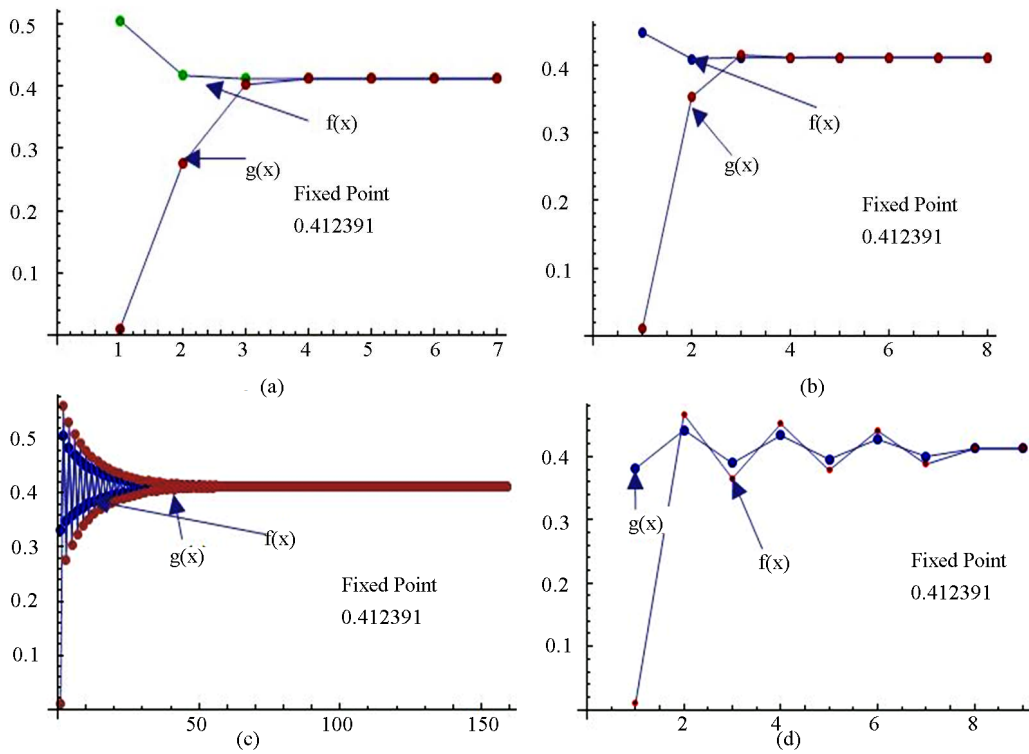


Figure 7. Graphical observations of simple Noor iteration for $p_1(x)$. Here (a)-(d) show the graph for Table 7. The merging point with value 0.412391 is fixed point.

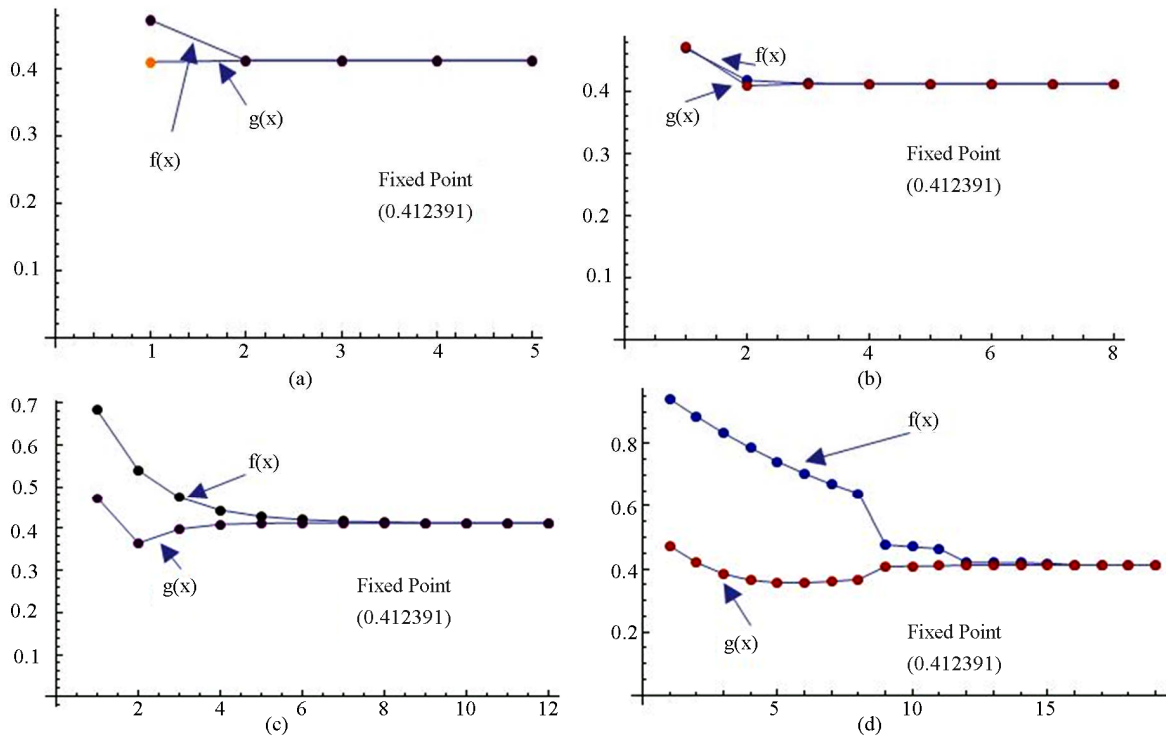


Figure 8. Graphical observations of new modified Noor iteration for $p_1(x)$. Here (a)-(d) show the graph for Table 8. The merging point with value 0.412391 is fixed point.

Table 9. Simple Ishikawa for $p_2(x)$.

N	$a = 0.9, b = 0.999$		$a = 0.1, b = 0.9$		$a = 0.3, b = 0.5$		$a = 0.1, b = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.340948	0.342793	0.343325	0.342793	0.346084	0.342793	0.349258	0.342793
1	0.340071	0.340243	0.341078	0.340895	0.343689	0.341668	0.348571	0.342578
2	0.339989	0.340005	0.340338	0.340278	0.342231	0.340996	0.347934	0.342379
3	0.339982	0.339983	0.340097	0.340078	0.341345	0.340593	0.347344	0.342195
6	0.339981	0.339981	0.339985	0.339984	0.340284	0.340116	0.345821	0.341726
7	0.339981	0.339981	0.339982	0.339982	0.340164	0.340063	0.345386	0.341593
9			0.339981	0.339981	0.340048	0.340011	0.344611	0.341357
10			0.339981	0.339981	0.340022	0.339999	0.344266	0.341253
18					0.339982	0.339981	0.342284	0.340659
20					0.339981	0.339981	0.341952	0.340561
21					0.339981	0.339981	0.341805	0.340517
127							0.339982	0.339981
128							0.339981	0.339981
129							0.339981	0.339981

Table 10. Modified Ishikawa for $p_2(x)$.

N	$a = 0.9, b = 0.9999$		$a = 0.1, b = 0.9$		$a = 0.3, b = 0.6$		$a = 0.1, b = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.33998	0.339823	0.340869	0.339823	0.343944	0.339823	0.348985	0.339823
1	0.339981	0.339981	0.340069	0.33998	0.341559	0.339957	0.348075	0.339854
2	0.339981	0.339981	0.33999	0.339981	0.340611	0.339977	0.347257	0.339879
4			0.339981	0.339981	0.340082	0.339981	0.345864	0.339915
5			0.339981	0.339981	0.340021	0.339981	0.345271	0.339928
---					-----	-----		
9					0.339982	0.339981	0.343443	0.339959
10					0.339981	0.339981	0.343096	0.339963
11					0.339981	0.339981	0.342783	0.339966
							-----	-----
93							0.339982	0.339981
94							0.339981	0.339981
95							0.339981	0.339981

Table 11. Simple Agarwal *et al.* for $p_2(x)$.

N	$a = 0.9, b = 0.99999,$		$a = 0.1, b = 0.9$		$a = 0.3, b = 0.5$		$a = 0.1, b = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.340949	0.342793	0.342605	0.342793	0.342481	0.342793	0.342772	0.342793
1	0.340071	0.340243	0.340649	0.340696	0.340587	0.340662	0.340737	0.340742
2	0.339989	0.340005	0.34015	0.340162	0.340127	0.340145	0.340184	0.340185
5	0.339981	0.339981	0.339984	0.339984	0.339983	0.339983	0.339985	0.339985
6	0.339981	0.339981	0.339982	0.339982	0.339982	0.339982	0.339982	0.339982
7	0.339981	0.339981	0.339981	0.339981	0.339981	0.339981	0.339981	0.339981
8			0.339981	0.339981	0.339981	0.339981	0.339981	0.339981

Table 12. Modified Aggarwal *et al.* for $p_2(x)$.

N	$a = 0.99999, b = 0.999$		$a = 0.1, b = 0.9$		$a = 0.1, b = 0.1$		$a = 0.1, b = 0.5$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.339981	0.339823	0.339851	0.339823	0.339827	0.339823	0.339839	0.339823
1	0.339981	0.339981	0.339981	0.339981	0.339981	0.339981	0.339981	0.339981
2	0.339981	0.339981	0.339981	0.339981	0.339981	0.339981	0.339981	0.339981

Table 13. Simple SP for $p_2(x)$.

N	$a = 0.9, b = 0.9, c = 0.999$		$a = 0.1, b = 0.1, c = 0.9$		$a = 0.3, b = 0.5, c = 0.7$		$a = 0.1, b = 0.1, c = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.340313	0.342793	0.343012	0.342793	0.342455	0.342793	0.347988	0.342793
1	0.339992	0.340071	0.340881	0.340808	0.340583	0.340655	0.346375	0.342211
2	0.339981	0.339984	0.340247	0.340225	0.340127	0.340144	0.345085	0.34175
3	0.339981	0.339981	0.340059	0.340053	0.340016	0.34002	0.344053	0.341386
4	0.339981	0.339981	0.340004	0.340002	0.33999	0.339991	0.343229	0.341097
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8			0.339981	0.339981	0.339981	0.339981	0.341292	0.340428
9			0.339981	0.339981	0.339981	0.339981	0.341026	0.340337
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44							0.339981	0.339981
45							0.339981	0.339981

Table 14. Modified SP for $p_2(x)$.

N	$a = 0.9, b = 0.9, c = 0.999$		$a = 0.1, b = 0.1, c = 0.9$		$a = 0.3, b = 0.5, c = 0.7$		$a = 0.1, b = 0.1, c = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.340066	0.339823	0.34807	0.339823	0.343427	0.339823	0.34807	0.339823
1	0.339982	0.339981	0.346515	0.339879	0.34118	0.339963	0.346515	0.339879
2	0.339981	0.339981	0.345263	0.339915	0.3404	0.339979	0.345263	0.339915
3	0.339981	0.339981	0.344252	0.339938	0.340128	0.339981	0.344252	0.339938
--	--	-----	-----	---	-----	-----	-----	-----
8			0.341463	0.339976	0.339982	0.339981	0.341463	0.339976
9			0.341181	0.339978	0.339981	0.339981	0.341181	0.339978
10			0.340953	0.339979	0.339981	0.339981	0.340953	0.339979
--	--	-----	-----	---	-----	-----	-----	-----
46			0.339982	0.339981			0.339982	0.339981
47			0.339981	0.339981			0.339981	0.339981
48			0.339981	0.339981			0.339981	0.339981

Table 15. Simple Noor for $p_2(x)$.

N	$a = 0.9, b = 0.9, c = 0.999$		$a = 0.1, b = 0.1, c = 0.9$		$a = 0.3, b = 0.5, c = 0.7$		$a = 0.1, b = 0.1, c = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.340491	0.342793	0.34332	0.342793	0.344161	0.342793	0.349258	0.342793
1	0.340006	0.340119	0.341074	0.340894	0.341717	0.341127	0.34857	0.342577
2	0.339982	0.339988	0.340337	0.340277	0.3407	0.340452	0.347932	0.342378
3	0.339981	0.339981	0.340097	0.340077	0.340279	0.340176	0.347342	0.342195
4	0.339981	0.339981	0.340019	0.340012	0.340104	0.340061	0.346794	0.342025
5	0.339981	0.339981	0.339993	0.339991	0.340032	0.340014	0.346288	0.341869
6			0.339985	0.339984	0.340002	0.339995	0.345818	0.341725
7			0.339982	0.339982	0.33999	0.339987	0.345383	0.341592
8			0.339981	0.339981	0.339985	0.339983	0.344981	0.34147
9			0.339981	0.339981	0.339983	0.339982	0.344608	0.341357
10					0.339982	0.339981	0.344263	0.341252
11					0.339981	0.339981	0.343943	0.341156
12					0.339981	0.339981	0.343647	0.341067
13					0.339981	0.339981	0.343373	0.340984
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127							0.339982	0.339981
128							0.339981	0.339981
129							0.339981	0.339981

Table 16. Modified Noor for $p_2(x)$.

N	$a = 0.9, b = 0.9, c = 0.999$		$a = 0.1, b = 0.1, c = 0.9$		$a = 0.3, b = 0.5, c = 0.7$		$a = 0.1, b = 0.1, c = 0.1$	
	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n	x_{n+1}	Tx_n
0	0.340982	0.339823	0.348985	0.339823	0.344953	0.339823	0.348985	0.339823
1	0.340081	0.33998	0.348075	0.339854	0.342458	0.339943	0.348075	0.339854
2	0.339991	0.339981	0.347257	0.339879	0.341217	0.339972	0.347257	0.339879
3	0.339982	0.339981	0.346523	0.339899	0.340599	0.339979	0.346523	0.339899
4	0.339981	0.339981	0.345864	0.339915	0.34029	0.33998	0.345864	0.339915
5	0.339981	0.339981	0.345271	0.339928	0.340135	0.339981	0.345271	0.339928
13			0.342249	0.339972	0.339982	0.339981	0.342249	0.339972
14			0.342022	0.339973	0.339981	0.339981	0.342022	0.339973
15			0.341817	0.339975	0.339981	0.339981	0.341817	0.339975
93			0.339982	0.339981			0.339982	0.339981
94			0.339981	0.339981			0.339981	0.339981

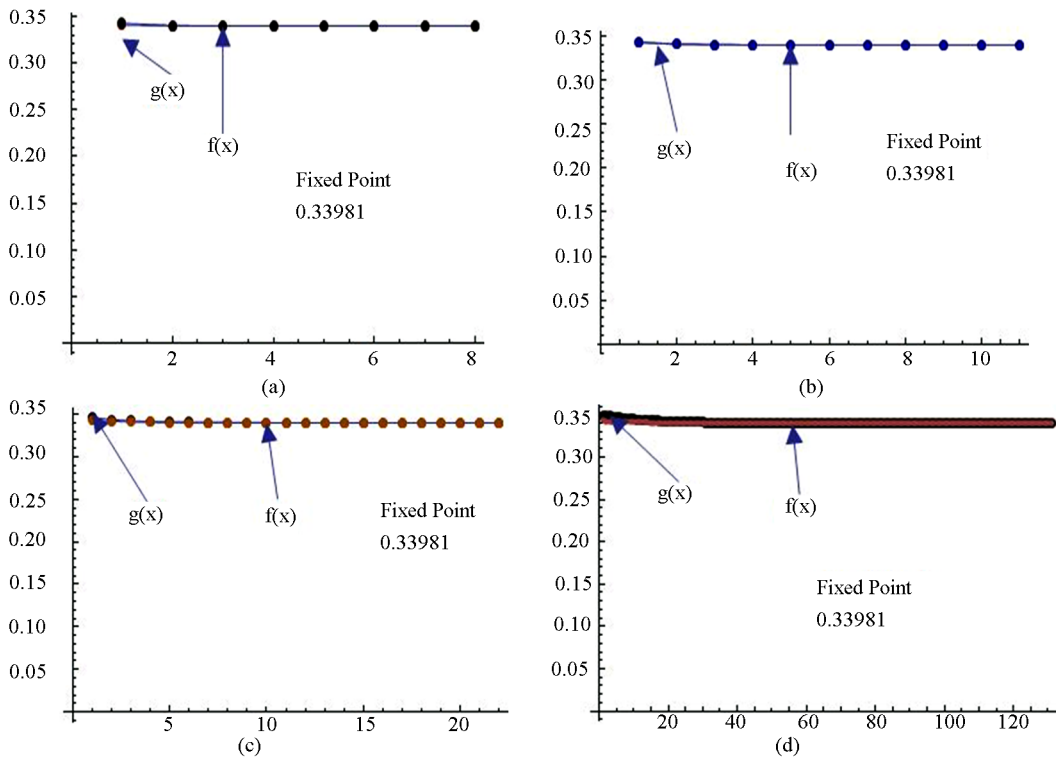


Figure 9. Graphical observations for simple Ishikawa iteration for $p_2(x)$. Here (a)-(d) show the graph for Table 9. The merging point with value 0.33981 is fixed point.

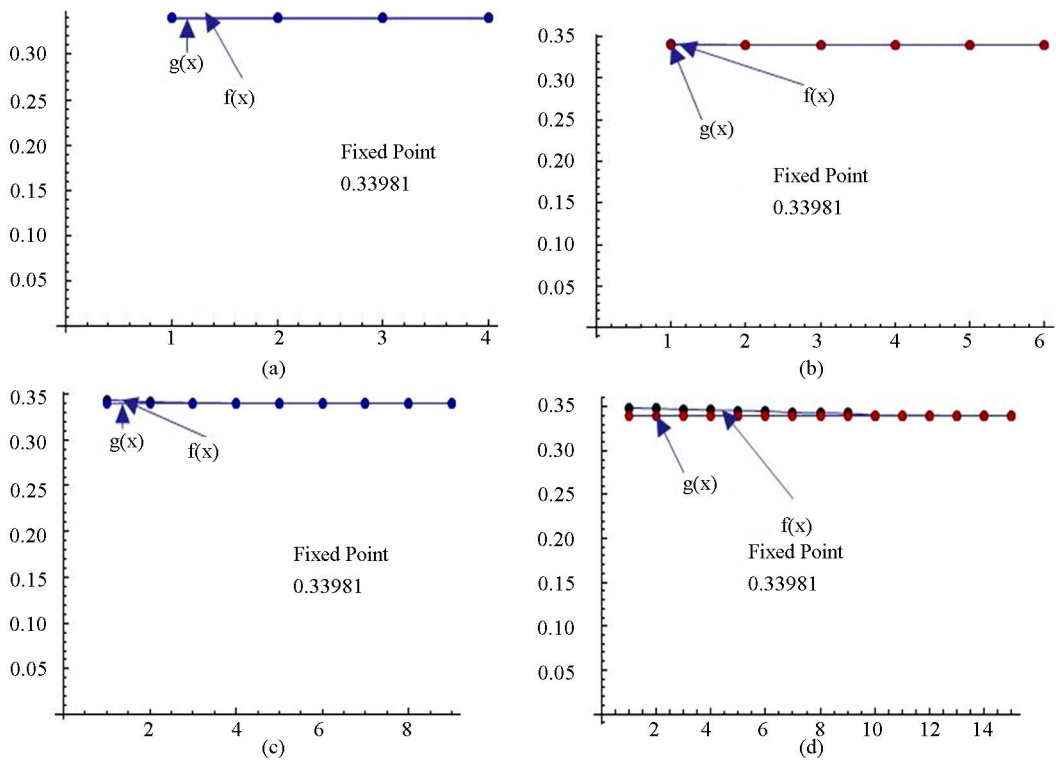


Figure 10. Graphical observations for new modified Ishikawa iteration for $p_2(x)$. Here (a)-(d) show the graph for Table 10. The merging point with value 0.33981 is fixed point.

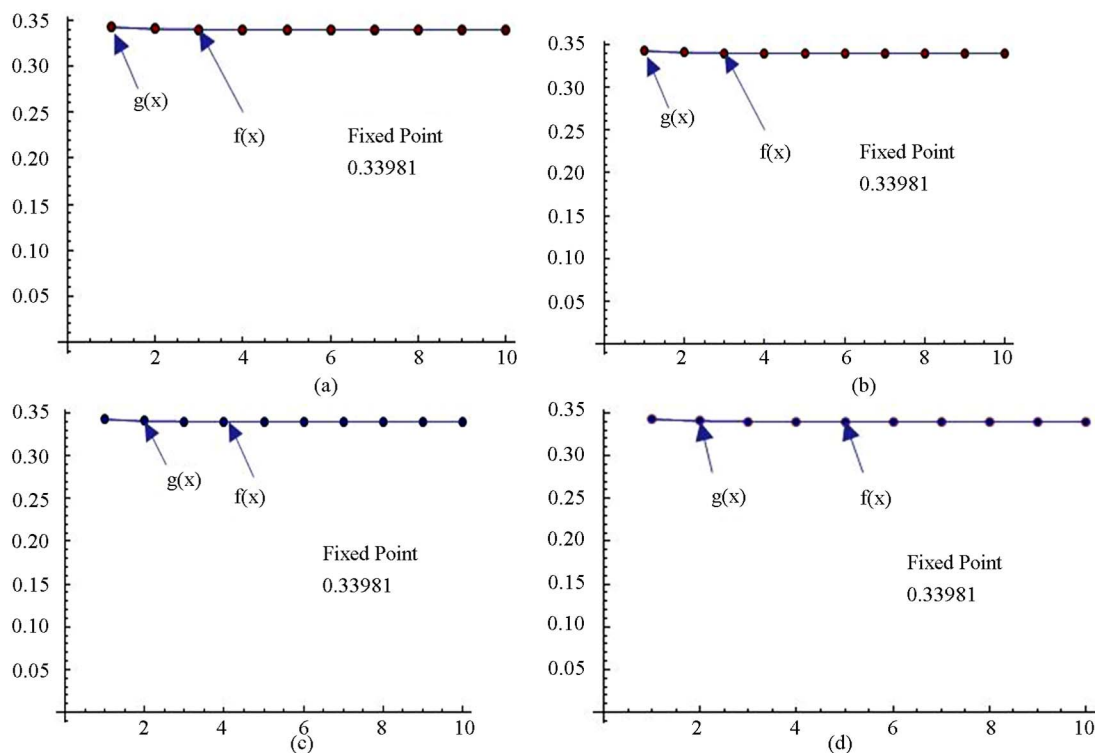


Figure 11. Graphical observations for simple Agarwal iteration for $p_2(x)$. Here (a)-(b) show the graph for Table 11. The merging point with value 0.33981 is fixed point.

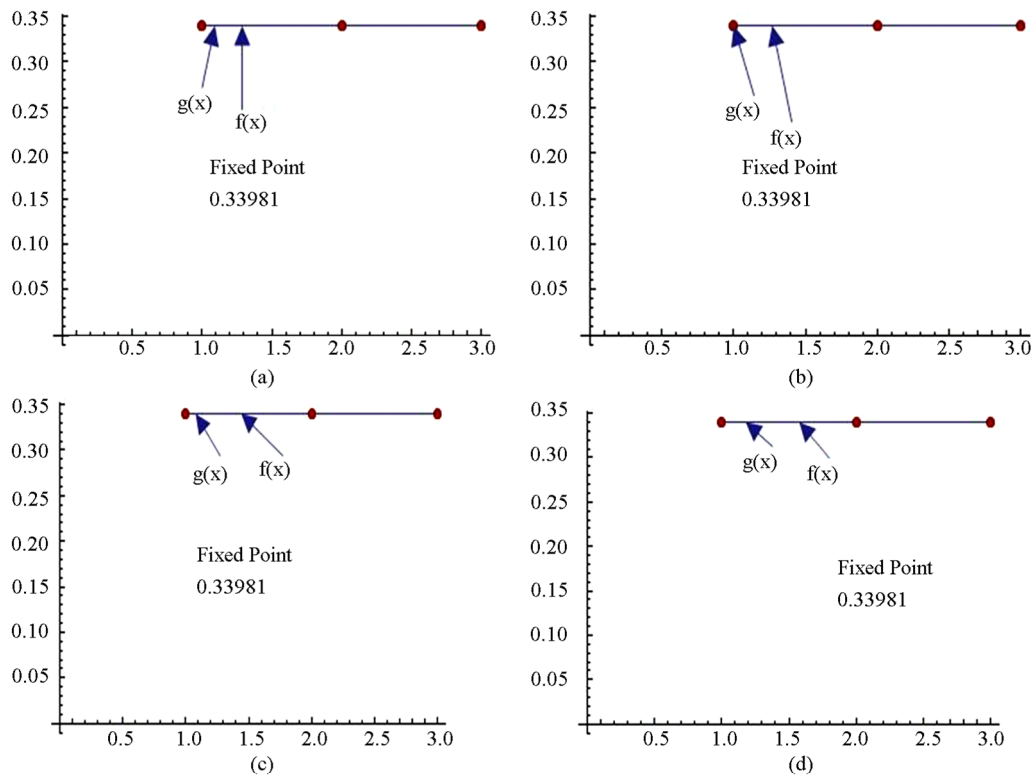


Figure 12. Graphical observations for simple Agarwal iteration for $p_2(x)$. Here (a)-(b) show the graph for Table 12. The merging point with value 0.33981 is fixed point.

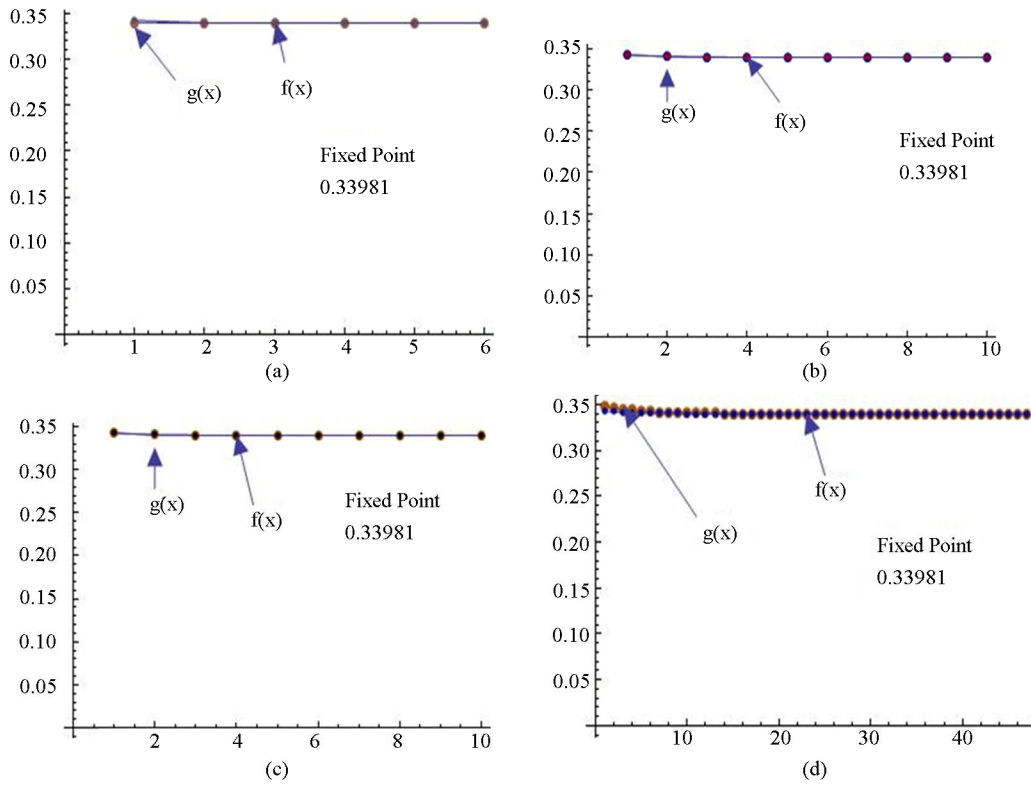


Figure 13. Graphical observations for simple SP iteration for $p_2(x)$. Here (a)-(b) show the graph for Table 13. The merging point with value 0.33981 is fixed point.

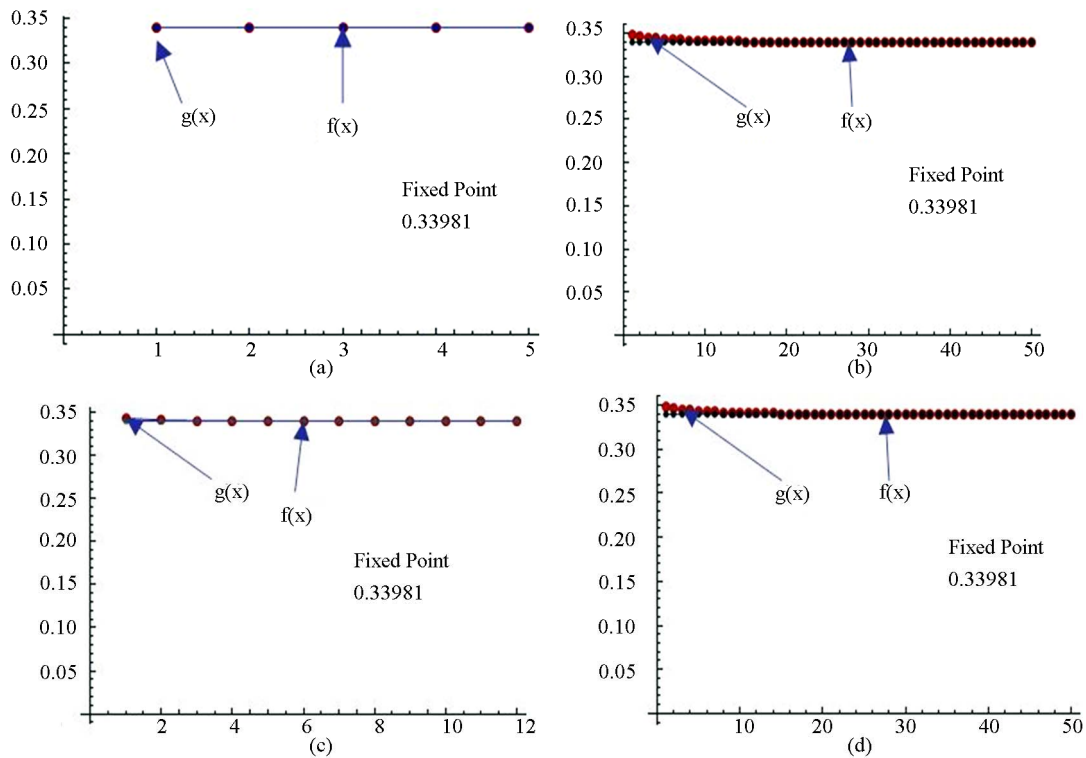


Figure 14. Graphical observations for new modified SP iteration for $p_2(x)$. Here (a)-(b) show the graph for Table 14. The merging point with value 0.33981 is fixed point.

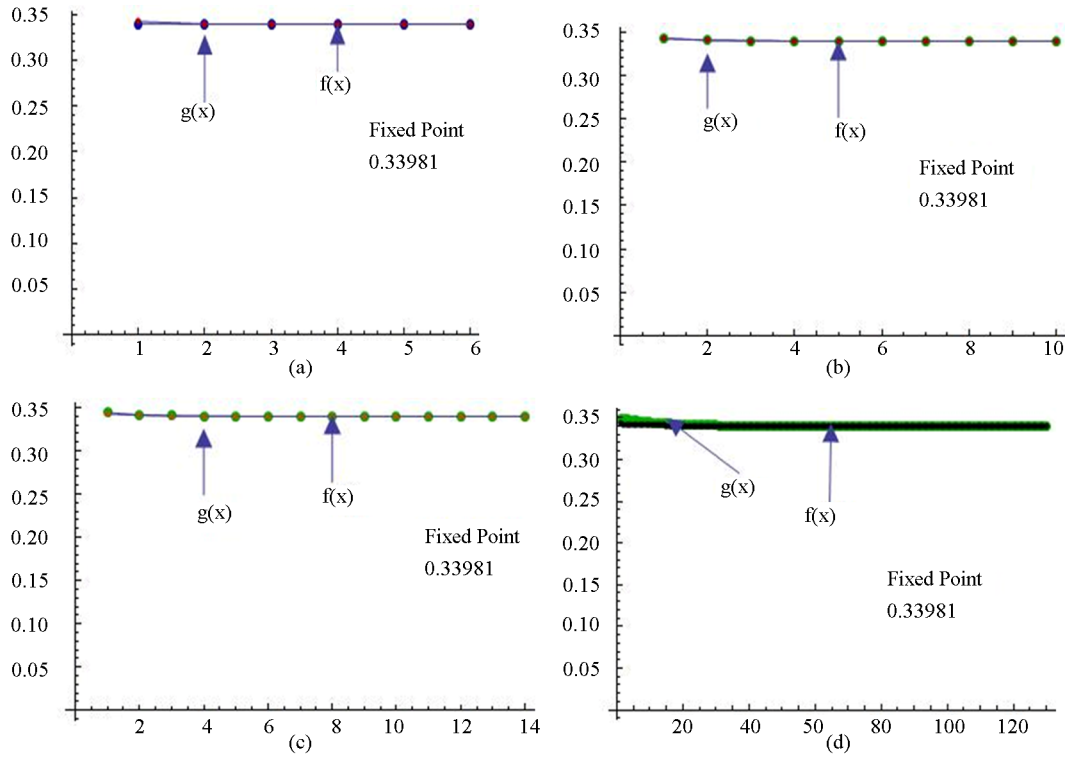


Figure 15. Graphical observations for simple Noor iteration for $p_2(x)$. Here (a)-(b) show the graph for Table 15. The merging point with value 0.33981 is fixed point.

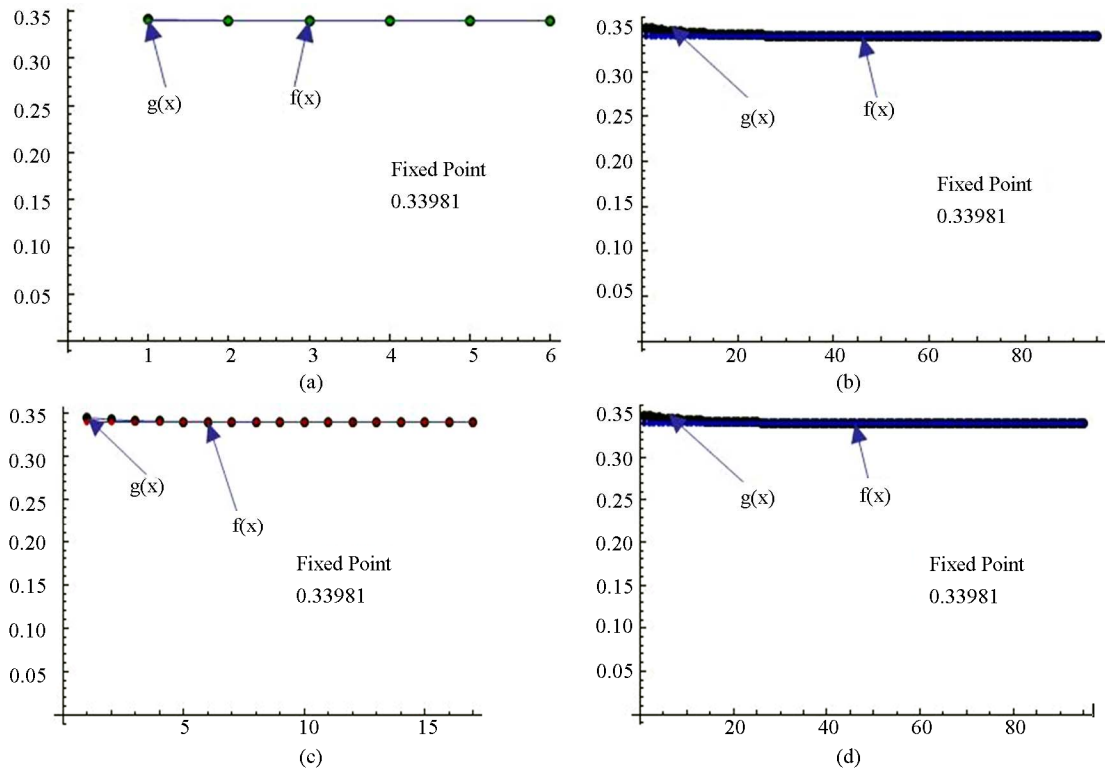


Figure 16. Graphical observations for new modified Noor iteration for $p_2(x)$. Here (a)-(b) show the graph for Table 16. The merging point with value 0.33981 is fixed point.

4. Observations

Table 17. Simple-Ishikawa for $p_1(x)$.

a/b	0.001	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	0.999	0.9999
0.001	5513	546	49	16	11	6	13	24	86	not	not	not	not	Not
0.01	5590	554	50	22	12	7	12	24	72	not	not	not	not	not
0.1	6503	645	59	26	15	9	8	12	24	53	3476	not	not	Not
0.2	7994	794	74	34	20	13	8	6	11	17	28	50	54	54
0.3	10455	1040	99	46	29	20	14	10	7	7	9	13	13	13
0.4	15285	1523	147	70	45	32	24	18	14	12	10	9	9	9
0.5	29149	2909	285	140	91	66	52	42	34	28	21	23	22	22
0.6	916911	91684	9161	4577	3048	2284	1826	1520	1301	1137	1009	916	907	906
0.7	not	not	not	not	not	not	not	not	not	not	not	not	not	not
0.8	not	not	not	not	not	not	not	not	not	not	not	not	not	not
0.9	not	not	not	not	not	not	not	not	not	not	not	not	not	not
0.99	not	not	not	not	not	not	not	not	not	not	not	not	not	not
0.999	not	not	not	not	not	not	not	not	not	not	not	not	not	not

Table 18. Modified-Ishikawa for $p_1(x)$.

a/b	0.001	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	0.999	0.9999
0.001	14030	1397	134	63	40	28	21	16	12	9	6	4	4	4
0.01	14032	1397	134	63	40	28	21	16	12	9	6	4	4	4
0.1	14056	1400	134	63	40	28	21	16	12	9	6	5	4	4
0.2	14086	1403	134	64	40	28	21	16	12	9	5	5	4	4
0.3	14119	1406	134	64	40	28	21	16	12	9	5	5	4	4
0.4	14152	1409	135	64	40	28	21	16	12	9	6	4	4	4
0.5	14181	1412	135	64	40	28	21	16	12	9	6	4	4	4
0.6	14204	1414	135	64	40	28	21	16	12	9	6	4	4	4
0.7	14220	1416	136	64	40	28	21	16	12	9	7	4	4	4
0.8	14224	1416	136	64	40	28	21	16	12	9	7	3	3	3
0.9	14216	1416	135	64	40	28	21	16	12	9	7	4	3	3
0.99	14197	1414	135	64	40	28	21	16	12	9	7	4	3	3
0.999	14195	1414	135	64	40	28	21	16	12	9	7	4	3	3

Table 19. Simple-Ishikawa for $p_1(x)$.

a/b	0.001	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	0.999	0.9999
0.001	13701	1366	132	65	41	29	23	18	15	12	10	8	8	8
0.01	13667	1363	132	64	41	29	22	18	14	12	10	8	8	8
0.1	13332	1329	129	62	40	29	22	17	14	11	9	8	8	8

Continued

0.2	12998	1296	125	60	39	28	21	17	13	11	9	8	7	7
0.3	12671	1263	122	59	38	27	20	16	13	11	9	7	7	7
0.4	12360	1232	119	57	36	26	20	16	12	10	8	7	7	7
0.5	12066	1203	116	56	36	25	19	15	12	10	8	7	6	6
0.6	11785	1174	113	54	35	25	19	15	12	9	8	6	6	6
0.7	11516	1148	110	53	34	24	18	14	11	9	7	6	6	6
0.8	11260	1122	108	52	33	23	18	14	11	8	7	6	6	5
0.9	11015	1096	106	50	32	23	17	13	10	8	7	5	5	5
0.99	10803	1076	104	49	31	22	17	13	10	8	6	5	5	5
0.999	10783	1074	103	49	31	22	17	13	10	8	6	5	5	5

Table 20. Modified-Ishikawa for $p_2(x)$.

a/b	0.001	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	0.999	0.9999
0.001	9977	994	95	45	28	20	15	11	9	7	5	3	2	2
0.01	9977	994	95	45	28	20	15	11	9	7	5	2	2	2
0.1	9980	994	95	45	28	20	15	11	9	7	5	2	2	2
0.2	9983	994	95	45	28	20	15	11	9	7	5	2	2	2
0.3	9985	994	95	45	28	20	15	11	9	7	5	2	2	2
0.4	9987	995	95	45	29	20	15	11	9	7	5	3	2	2
0.5	9988	995	95	45	29	20	15	11	9	7	5	3	2	2
0.6	9990	995	95	45	29	20	15	11	9	7	5	3	2	2
0.7	9991	995	95	45	29	20	15	11	9	7	5	3	2	2
0.8	9992	995	95	45	29	20	15	11	9	7	5	3	2	2
0.9	9992	995	95	45	29	20	15	11	9	7	5	3	2	2
0.99	9992	995	95	45	29	20	15	11	9	7	5	3	2	2
0.999	9992	995	95	45	29	20	15	11	9	7	5	3	2	2

Table 21. Simple-Agarwal for $p_1(x)$.

a/b	0.001	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	0.999	0.9999
0.001	not	not	not	not	not	not	Not	not	not	not	not	not	not	not
0.01	not	not	not	not	not	not	Not	not	not	not	not	not	not	not
0.1	not	not	not	not	not	not	Not	not	not	not	not	not	not	not
0.2	not	not	not	not	not	not	Not	not	not	291	97	59	57	57
0.3	not	not	not	not	not	not	5475	101	47	29	19	14	14	14
0.4	not	not	not	not	not	307	59	29	19	11	8	9	10	10
0.5	not	not	not	not	not	59	27	15	9	9	15	23	24	24
0.6	not	not	not	not	101	29	15	7	10	17	37	288	741	885
0.7	not	not	not	not	47	18	9	8	20	62	not	not	not	not
0.8	not	not	not	287	29	11	8	19	68	not	not	not	not	not
0.9	not	not	not	95	17	7	15	53	not	not	not	not	not	not
0.99	not	not	not	55	14	10	33	not	not	not	not	not	not	not
0.999	not	not	not	53	14	10	39	not	not	not	not	not	not	not

Table 22. Modified-Agarwal for $p_1(x)$.

a/b	0.001	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	0.999	0.9999
0.001	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.01	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.1	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.2	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.3	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.5	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.6	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.7	4	4	4	4	4	4	4	4	4	4	4	4	4	4
0.8	4	4	4	4	4	4	4	4	4	3	3	3	3	3
0.9	4	4	4	4	4	4	4	3	3	3	3	3	3	3
0.99	4	4	4	4	4	4	4	3	3	3	3	3	3	3
0.999	4	4	4	4	4	4	4	3	3	3	3	3	3	3

Table 23. Simple-Agarwal for $p_2(x)$.

a/b	0.001	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	0.999	0.9999
0.001	8	8	8	8	8	8	8	8	8	8	8	8	8	8
0.01	8	8	8	8	8	8	8	8	8	8	8	8	8	8
0.1	8	8	8	8	8	8	8	8	8	8	8	8	8	8
0.2	8	8	8	8	8	8	8	8	8	8	8	8	7	7
0.3	8	8	8	8	8	8	8	8	7	7	7	7	7	7
0.4	8	8	8	8	8	8	7	7	7	7	7	7	7	7
0.5	8	8	8	8	8	7	7	7	7	7	7	6	6	6
0.6	8	8	8	8	8	7	7	7	7	7	6	6	6	6
0.7	8	8	8	8	7	7	7	7	6	6	6	6	6	6
0.8	8	8	8	8	7	7	7	7	6	6	6	6	5	5
0.9	8	8	8	8	7	7	7	6	6	6	5	5	5	5
0.99	8	8	8	7	7	7	6	6	6	6	5	5	5	5
0.999	8	8	8	7	7	7	6	6	6	5	5	5	5	5

Table 24. Modified-Agarwal for $p_2(x)$.

a/b	0.001	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99	0.999	0.9999
0.001	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.01	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.1	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.3	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.4	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.5	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.6	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Continued

0.7	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.8	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.9	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.99	2	2	2	2	2	2	2	2	2	2	2	2	2	2
0.999	2	2	2	2	2	2	2	2	2	2	2	2	2	2

We have noted the converging step of different iterations in tabular form and compare the conversing step for different value of a, b, c. Now by comparative analysis we noted that

1) For $p_1(x)$, simple Ishikawa do not converge for $0 < a \leq 0.1$, $0.8 < b \leq 1$ and $0.6 < a < 1$, $0 < b < 1$ but new modified Ishikawa converges for all values of a and b converges faster than Ishikawa iteration for corresponding values of a, b. Also it converges at lesser step as a and b both approaches one but not so in case of simple Ishikawa as observe from **Tables 17** and **18**. Similarly if we compare the both iterations for $p_2(x)$ as observed from **Tables 19** and **20** that as we increase values of a and b simultaneously than converging step decreases for both iterations but modified Ishikawa iteration converges at lesser step for $p_2(x)$.

2) As observed from **Tables 21** and **22** for $p_1(x)$ simple Agarwal *et al.* do not converge for all values of a and b it converges for $\{a = 0.2, 0.8 \leq b < 1\}$, $\{a = 0.3, 0.5 \leq b < 1\}$, $\{a = 0.4, 0.5, 0.4 \leq b < 1\}$, $\{a = 0.6, 0.3 \leq b < 1\}$, $\{a = 0.7, 0.3 \leq b \leq 0.8\}$, $\{0.8 \leq a < 1, 0.2 \leq b \leq 0.5\}$

but modified new Agarwal *et al.* iteration converges at lesser step for all values of a, b. For $p_2(x)$ both iterations converge for all values of a and b but modified iteration converges faster than simple iteration

3) The simple SP iteration converges at lesser step for $p_1(x)$ when $a = 1/2$, $b = 1/2$, $c = 1/2$ as we increases a and b the step of convergence increases. But do not converge if a and b approaches to one whereas modified new SP iteration converges for all values of a, b, c and at lesser step than simple SP. For $p_2(x)$ both iteration converges for all values of a, b and c but modified new SP converge faster than Simple SP iteration.

4) For simple Noor and modified new Noor iteration result is same as for SP and modified new SP iterations.

5. Conclusion

By the observation formed from the program and graph

drawn in C++ and Mathematica for $p_1(x)$ and $p_2(x)$ polynomial, we conclude that the modified Ishikawa, Agarwal *et al.*, SP, Noor are faster than simple Ishikawa, Agarwal *et al.*, SP, Noor; but if we compare modified Ishikawa, Agarwal *et al.*, SP, Noor with decreasing order of rate of convergence of modified Agarwal *et al.*, SP, Noor, Ishikawa, modified new Agarwal *et al.* have consistent rate of convergence. The graphs drawn are based on data formed from C++ program and plot the data in mathematica to show the fixed point.

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