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Construction of Five-step Continuous Block General Method for the Solution of Ordinary Differential Equations

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Authors' contributions

This work was carried out in collaboration between all authors. Author DR wrote the draft of the manuscript. Author JAO and AL managed the literature searches. Author DR designed the figures, managed literature searches and contributed to the correction of the draft. Author JZD provided the case, the figures and supervised the work. All authors read and approved the final manuscript.

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ABSTRACT

In this paper, a self starting five step Continuous Block Hybrid Adams Moulton Method (CBHAM) with three off-grid points is developed using collocation and interpolation procedures. The predictor schemes are then expanded using Taylor's series expansion. Multiple numerical integrators were produced and arrived at some discrete schemes. The discrete schemes are of uniform order and are assembled into a single block matrix equation. These equations are simultaneously applied to provide the approximate solution for stiff initial value problems for ordinary differential equations. The order of accuracy and stability of the block method is discussed and its accuracy is established numerically.

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1. INTRODUCTION

Consider the first order initial value problems of the form

$$y^{'} = f(x,y), y(x_0) = y_0, x \in [a,b] \tag{1}$$

The solution is in the range $a \le x \le b$, where a and b are finite and we assumed that f satisfies Lipschitz condition, which guarantees the existence and uniqueness of solution of the problem (1).

The discrete solution of (1) by linear multi-step methods has being studied by authors like [1] One important advantage of the continuous over discrete approach is the ability to provide discrete schemes for simultaneous integration

These discrete schemes can as well be reformulated as general linear method (GLM) [2].

Many researchers have worked on the development of continuous linear multi-step method in finding solution to (1). These scholars proposed methods with different basis functions, among them are, [3,4,5], to mention few. These block methods are self- starting and can directly be applied to stiff problems.

In this paper, we present the construction of five step continuous block general linear method with three off-grid points. The derived schemes will be applied in a block form.

2. DERIVATION OF THE METHODS

We define the K-step continuous Hybrid formula to be of the form

$$y(x) = \sum_{j=1}^{r-1} \alpha_j(x) y_{n+j} + \sum_{j=1}^{s-1} h_n \beta_j(x) f_{n+j} \quad , \quad n = 0, k, 2k, ..., j \tag{2}$$

Where t and s denote the number of interpolation and collocation points respectively and h_n the variable step-size which is valid in the k-step, $x_n \le x \le x_{n+k}$. Note that,

$$\alpha_i(x) = \sum_{r=1}^{t} C_{ri}\phi_r(x) + \sum_{r=t+1}^{p} C_{ri}\phi_r(x) \tag{3}$$

$$h_n\beta_i(x) = \sum_{r=1}^{t} C_{ri}\phi_r(x) + \sum_{r=t+1}^{p} C_{ri}\phi_r(x) \tag{4}$$

The polynomial $\phi_1,...,\phi_p$ are given bases and $p = t + s - 1$ is the degree of the polynomial interpolation Y and the collocation points $C_i, i = 1,...,s$ and C_{ij} are element of an inverse matrix C , for the initial value problems given in the form of (1).

The formulas in equations (2), (3) and (4) are obtained from the multi-step collocation following [6] which was a generalization of [1]. The expansion of

$$y(x) \approx Y(x) = \alpha_1\phi_1(x) + \alpha_2\phi_2(x) + ... + \alpha_p\phi_p(x) , \quad x_n \le x \le x_{n+k} \tag{5}$$

Starting with (5) and imposing the following conditions

$$\begin{Bmatrix} \alpha_1\phi_1(x_j) + ... + \alpha_p\phi_p(x_j) = y_j, j = 1,...,t \\ \alpha_1\phi'(x_j) + ... + \alpha_p\phi'(x_j) = f_i, i = 1,...,s \end{Bmatrix} \tag{6}$$

Putting (6) in matrix form, we have

$$CA = I \tag{7}$$

where is the identity matrix of appropriate dimension

$$A=\begin{pmatrix} 1 & x_n & x_n^2\cdots x_n^t & \cdots x_n^{t+m-1} \\ 1 & x_{n+1} & x_{n+1}^2\cdots x_{n+1}^t & \cdots x_{n+1}^{t+m-1} \\ \vdots & \vdots & \vdots\ \ddots\ \vdots & \ddots\ \vdots \\ 1 & x_{n+t-1} & x_{n+t-1}^2\cdots x_{n+t-1}^t\cdots x_{n+t-1}^{t+m-1} \\ 0 & 1 & 2\bar{x}_0\cdots t x_1^{(t-1)}\cdots(t+m-1)\bar{x}_1^{t+m-2} \\ \vdots & \vdots & \vdots\ \ddots\ \vdots\ \ \ddots\ \vdots \\ 0 & 1 & 2\bar{x}_m\cdots t\bar{x}_m^{(t-1)}\ \cdots(t+m-1)\bar{x}_m^{t+m-2} \end{pmatrix} \tag{8}$$

and

$$C=\begin{pmatrix} C_{1,1} & C_{1,2} & \cdots & C_{1,t} & C_{1,t+1} & \cdots & C_{1,t+s} \\ C_{2,1} & C_{2,2} & \cdots & C_{2,t} & C_{2,t+1} & \cdots & C_{2,t+s} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ C_{t,1} & C_{t,2} & \cdots & C_{t,t} & C_{t,t+1} & \cdots & C_{t,t+s} \\ C_{t+1,1} & C_{t+1,2} & \cdots & C_{t+1,t} & C_{t+1,t+1} & \cdots & C_{t+1,t+s} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ C_{t+s,1} & C_{t+s,2} & \cdots & C_{t+s,t} & C_{t+s,t+1} & \cdots & C_{t+s,t+s} \end{pmatrix} \tag{9}$$

We call A the multi-step collocation and interpolation matrix which has a very simple structure and of dimension *(t+m)* × *(t+m)*. As can be seen the entries of C are the constant coefficients of the polynomial given in (2) which are to be determined.

3. SPECIFICATION OF THE METHODS

$$A=\begin{bmatrix} 1 & x_n & x^2{}_n\ldots & x_n{}^{t+m-1} \\ 1 & x_{n+1} & x^2{}_{n+2} & x_n{}^{t+m-1} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 1 & x_{n+t-1} & x^2{}_{n+t-1} & x^{t+m-1}{}_{n+t-1} \\ 0 & 1 & 2\bar{x}_0 & (t+m-1)\bar{x}_0{}^{t+m-2} \\ 0 & 1 & 2\bar{x}_0 & (t+m-1)\bar{x}_1{}^{t+m-2} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 0 & 1 & 2\bar{x}_{m-1} & (t+m-1)\bar{x}_{m-1}{}^{t+m-2} \end{bmatrix} \tag{10}$$

The parameters required for equation (10) are k=5, t=1, m=k+3

$$(x_n, x_{n+5}),\left(\bar{x}_0 = x_n, \bar{x}_1 = x_{n+1}, \bar{x}_{3/2} = x_{n+3/2}, \bar{x}_2 = x_{n+2}, \bar{x}_{5/2} = x_{n+5/2}, \bar{x}_3 = x_{n+3}, \bar{x}_{7/2} = x_{n+7/2}, \bar{x}_4 = x_{n+4}, \bar{x}_5 = x_{n+5}\right).$$

3.1 The Continuous General Block Method: Case K=5, with 3 Off-grid Points

The collocation matrix A is given by

$$\begin{pmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & x_n^9 \\ 0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 & 6x_n^5 & 7x_n^6 & 8x_n^7 & 9x_n^8 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^4 & 6x_{n+1}^5 & 7x_{n+1}^6 & 8x_{n+1}^7 & 9x_{n+1}^8 \\ 0 & 1 & 2x_{n+\frac{3}{2}} & 3x_{n+\frac{3}{2}}^2 & 4x_{n+\frac{3}{2}}^3 & 5x_{n+\frac{3}{2}}^4 & 6x_{n+\frac{3}{2}}^5 & 7x_{n+\frac{3}{2}}^6 & 8x_{n+\frac{3}{2}}^7 & 9x_{n+\frac{3}{2}}^8 \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^4 & 6x_{n+2}^5 & 7x_{n+2}^6 & 8x_{n+2}^7 & 9x_{n+2}^8 \\ 0 & 1 & 2x_{n+\frac{5}{2}} & 3x_{n+\frac{5}{2}}^2 & 4x_{n+\frac{5}{2}}^3 & 5x_{n+\frac{5}{2}}^4 & 6x_{n+\frac{5}{2}}^5 & 7x_{n+\frac{5}{2}}^6 & 8x_{n+\frac{5}{2}}^7 & 9x_{n+\frac{5}{2}}^8 \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^2 & 4x_{n+3}^3 & 5x_{n+3}^4 & 6x_{n+3}^5 & 7x_{n+3}^6 & 8x_{n+3}^7 & 9x_{n+3}^8 \\ 0 & 1 & 2x_{n+\frac{7}{2}} & 3x_{n+\frac{7}{2}}^2 & 4x_{n+\frac{7}{2}}^3 & 5x_{n+\frac{7}{2}}^4 & 6x_{n+\frac{7}{2}}^5 & 7x_{n+\frac{7}{2}}^6 & 8x_{n+\frac{7}{2}}^7 & 9x_{n+\frac{7}{2}}^8 \\ 0 & 1 & 2x_{n+4} & 3x_{n+4}^2 & 4x_{n+4}^3 & 5x_{n+4}^4 & 6x_{n+4}^5 & 7x_{n+4}^6 & 8x_{n+4}^7 & 9x_{n+4}^8 \\ 0 & 1 & 2x_{n+5} & 3x_{n+5}^2 & 4x_{n+5}^3 & 5x_{n+5}^4 & 6x_{n+5}^5 & 7x_{n+5}^6 & 8x_{n+5}^7 & 9x_{n+5}^8 \end{pmatrix} \tag{11}$$

Similarly, we invert the matrix A in equation (1) above , which leads to the following Continuous scheme.

$$y(x) = \alpha_0 y_{n+1} + h\begin{bmatrix} \beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_{3/2}(x)f_{n+3/2} + \beta_2(x)f_{n+2} + \beta_{5/2}(x)f_{n+5/2} + \beta_3(x)f_{n+3} + \beta_{7/2}(x)f_{n+7/2} \\ + \beta_4(x)f_{n+4} + \beta_5(x)f_{n+5} \end{bmatrix} \tag{12}$$

where

$\alpha_0 = 1.$

$\beta_0 = \frac{1}{2268000}x^3\Big(1344x_n^6 + 4183h^6 + 22680x_n^5 h + 154980x_n^4 h^2 + 548100x_n^3 h^4 + 1048950x_n h^5 - 20160xx_n^5 - 2622375xh^5 + 1056132x^2h^4 + 20160x^2x_n^4 - 274050x^3h^3 - 13440x^3x_n^3 + 44280x^4h^2 + 5760x^4x_n^2 - 4050x^5h - 1440x^5x_n - 283500xx_n^4 h - 1549800xx_n^3 h^2 - 4110750xx_n^2 h^3 - 5280660xx_n h^4 + 1644300x^2x_n h^3 + 929880x^2x_n^2 h^2 + 226800x^2x_n^3 h - 309960x^3x_n h^2 - 113400x^3x_n^2 h + 32400x^4x_n h + 160x^6\Big)/h^8$

$\beta_1 = \frac{-1}{453600}x^3\Big(94080x_n^6 + 13948200h^6 + 1517040x_n^5 h + 9765000x_n^4 h^2 + 31857000x_n^3 h^3 + 54815040x_n^2 h^4 + 46018980x_n h^5 - 141120xx_n^5 - 11504745xh^5 + 5481504x^2h^{12} - 1592850x^3h^{11} + 279000x^4h^2 + 40320x^4x_n^2 - 27090x^5h^9 - 1896300xx_n^4 h - 9765000xx_n^3 h^2 - 23892750xx_n^2 h^3 - 27407520xx_n h^4 + 9557100x^2x_n h^{11} + 5859000x^2h^{10}x_n^2 + 1517040x^2h^9x_n^3 + 141120x^2h^8x_n^4 - 1953000x^3h^{10}x_n - 758520x^3h^9x_n^2 - 94080x^3h^8x_n^3 + 216720x^4x_n h - 10080x^5h^8x_n + 1120x^6\Big)/h^8$

$\beta_2 = \frac{2}{14175}x^3\Big(6720x_n^6 + 748200h^6 + 105840x_n^5 h + 661500x_n^4 h^2 + 2079000x_n^3 h^3 + 3409560x_n^2 h^4 + 2687580x_n h^5 - 10080xx_n^5 - 671895xh^5 + 18900x^4h^2 + 2880x^4x_n - 132300xx_n^4 h - 661500xx_n^3 h^2 -$

$1559250x\overset{2}{x^{n}}h^{3} - 1704780xx^{n}h^{4} + 15120x^{4}x^{n}h + 80x^{6} + 623700x^{2}x^{n}h^{3} + 396900x^{2}\overset{2}{x^{n}}h^{2} +$

$105840x^{2}\overset{3}{x^{n}}h - 132300x^{3}x^{n}h^{2} - 52920x^{3}\overset{2}{x^{n}}h + 340956x^{2}h^{4} + 10080x^{2}\overset{4}{x^{n}} - 103950x^{3}h^{3} -$

$6720x^{3}\overset{3}{x^{n}} - 1890x^{5}h - 720x^{5}x^{n}\Big)/h^{8}$

$\beta^{3} \quad = \quad \frac{-1}{45360}x^{3}\Big(94080\overset{6}{x^{n}} + 8297100h^{6} + 1446480\overset{5}{x^{n}}h + 8782200\overset{4}{x^{n}}h^{2} + 26657400\overset{3}{x^{n}}h +$

$41940360\overset{2}{x^{n}}h^{4} + 31486140x^{n}h^{5} - 141120x\overset{5}{x^{n}} - 7871535xh^{5} - 1808100x\overset{4}{x^{n}}h - 8782200x\overset{3}{x^{n}}h^{2} -$

$19993050x\overset{2}{x^{n}}h^{3} - 20970180xx^{n}h^{4} + 1120x^{6} + 7997220x^{2}x^{n}h^{3} + 5269320x^{2}\overset{2}{x^{n}}h^{2} +$

$1446480x^{2}\overset{3}{x^{n}}h - 1756440x^{3}x^{n}h^{2} - 723240x^{3}\overset{2}{x^{n}}h + 4194036x^{2}h^{4} + 141120x^{2}\overset{4}{x^{n}} - 1332870x^{3}h^{3} -$

$94080x^{3}\overset{3}{x^{n}} - 10080x^{5}x^{n} + 250920x^{4}h^{10} + 206640x^{4}h^{9}x^{n} + 40320x^{4}h^{8}\overset{2}{x^{n}}\Big)/h^{8}$

$\beta^{4} \quad = \quad \frac{4}{70875}x^{3}\Big(3424680h^{6} + 47040\overset{6}{x^{n}} + 705600\overset{5}{x^{n}}h + 4164300\overset{4}{x^{n}}h^{2} + 12243000\overset{3}{x^{n}}h^{3} +$

$18600120\overset{2}{x^{n}}h^{4} + 13463100x^{n}h^{5} - 70560x\overset{5}{x^{n}} - 3365775xh^{5} + 1860012x^{2}h^{4} + 70560x^{2}\overset{4}{x^{n}} -$

$612150x^{3}h^{3} - 47040x^{3}\overset{3}{x^{n}} + 118980x^{4}h^{2} + 20160x^{4}\overset{2}{x^{n}} - 5040x^{5}x^{n} - 882000x\overset{4}{x^{n}}h -$

$4164300x^{3}xh^{2} - 9182250x\overset{2}{x^{n}}h^{3} - 9300060xx^{n}h^{4} + 3672900x^{2}x^{n}h^{3} + 2498580x^{2}\overset{2}{x^{n}}h^{2} +$

$705600x^{2}\overset{3}{x^{n}}h - 832860x^{3}x^{n}h^{2} - 352800x^{3}\overset{2}{x^{n}}h + 100800x^{4}x^{n}h + 560x^{6}\Big)/h^{8}$

$\beta^{5} \quad = \quad \frac{-1}{45360}x^{3}\Big(94080\overset{6}{x^{n}} + 5825400h^{6} + 1375920\overset{5}{x^{n}}h + 7900200\overset{4}{x^{n}}h^{2} + 22566600\overset{3}{x^{n}}h^{3} +$

$33309360\overset{2}{x^{n}}h^{4} + 23462460x^{n}h^{5} - 141120x\overset{5}{x^{n}} - 5865615xh^{5} + 141120x^{2}\overset{4}{x^{n}} + 3330936x^{2}h^{4} -$

$1128330x^{3}h - 94080x^{3}\overset{3}{x^{n}} + 225720x^{4}h^{2} + 40320x^{4}\overset{2}{x^{n}} - 24570\overset{5}{x^{n}}h - 10080x^{5}x^{n} - 1719900x\overset{4}{x^{n}}h -$

$1692495x\overset{2}{x^{n}}h^{3} - 16654680xx^{n}h^{4} + 6769980x^{2}x^{n}h + 4740120x^{2}\overset{2}{x^{n}}h + 1375920x^{2}\overset{3}{x^{n}}h -$

$1580040x^{3}x^{n}h^{2} - 687960x^{3}\overset{2}{x^{n}}h + 196560x^{4}x^{n}h + 1120x^{6}\Big)/h^{8}$

$\beta^{6} \quad = \quad \frac{2}{14175}x^{3}\Big(361800h^{6} + 6720\overset{6}{x^{n}} + 95760\overset{5}{x^{n}}h + 535500\overset{4}{x^{n}}h^{2} + 1491000\overset{3}{x^{n}}h^{3} + 2149560\overset{2}{x^{n}}h^{4} +$

$1483020x^{n}h^{5} - 100800x\overset{5}{x^{n}} - 370755xh^{5} + 10080x^{2}\overset{4}{x^{n}} + 214956x^{2}h^{4} - 74550x^{3}h^{3} - 6720x^{3}\overset{3}{x^{n}} +$

$15300x^{4}h^{2} + 2880x^{4}\overset{2}{x^{n}} - 720x^{5}x^{n} - 1710x^{5}h + 80x^{6} - 119700x\overset{4}{x^{n}}h - 535500x\overset{3}{x^{n}}h^{2} -$

$118250x\overset{2}{x^{n}}h^{3} - 1074780xx^{n}h^{4} + 321300x^{2}\overset{2}{x^{n}}h^{2} + 95760x^{2}\overset{3}{x^{n}}h + 447300x^{2}x^{n}h^{3} -$

$107100x^{3}x^{n}h^{2} - 47880x^{3}\overset{2}{x^{n}}h + 13680x^{4}x^{n}h\Big)/h^{8}$

$\beta^{7} \quad = \quad \frac{-1}{453600}x^{3}\Big(94080\overset{6}{x^{n}} + 1305360\overset{5}{x^{n}}h + 7119000\overset{4}{x^{n}}h^{2} + 27410040\overset{2}{x^{n}}h^{3} + 27410040\overset{2}{x^{n}}h^{4} +$

$18606420x^{n}h^{5} - 141120x\overset{5}{x^{n}} - 4651605xh^{5} + 141120x^{2}h^{4} - 94080x^{3}\overset{3}{x^{n}} - 969150x^{3}h^{3} +$

$$40320x^4x_n^2 + 203400x^4h^2 - 23310x^5h - 10080x^5x_n + 4479300h^6 - 1631700xx_n^4h -$$
$$7119000xx_n^3h^2 - 14537250xx_n^2h^3 - 13705020xx_nh^4 + 1305360x^2x_n^3h + 4271400x^2x_n^2h^2 +$$
$$5814900x^2x_nh^3 - 652680x^3x_n^2h - 1423800x^3x_nh^2 + 186480x^4x_nh + 1120x^6\Big)/h^8$$

$$\beta^8 = \frac{1}{2268000}x^3\Big(13440x_n^6 + 176400x_n^5h + 919800x_n^4h^2 + 2415000x_n^3h^3 + 3316320x_n^2h^4 +$$
$$2198700x_nh^5 - 20160xx_n^5 - 549675xh^5 + 20160x^2x_n^4 + 331632x^2h^4 - 13440x^3x_n^3 - 120750x^3h^3 +$$
$$5760x^4x_n^2 + 26280x^4h^2 - 3150x^5h^9 + 519480h^6 - 220500xx_n^4h - 919800xx_n^3h^2 - 1811250xx_n^2h^3 -$$
$$165816xx_nh^4 + 176400x^2x_n^3h + 1551880x^2x_n^2h^2 + 724500x^2x_nh^3 - 88200x^3x_n^2h -$$
$$1839600x^3x_nh^2 + 25200x^4x_nh - 1440x^5h^8x_n + 160x^6\Big)/h^8$$

Evaluating the continuous scheme (12) at $x = x_{n+1}$, $x = x_{n+3/2}$, $x = x_{n+2}$, $x = x_{n+5/2}$, $x = x_{n+3}$, $x = x_{n+7/2}$, $x = x_{n+4}$, $x = x_{n+5}$, we obtain respectively the following order eight block hybrid method which is zero-stable and L-stable by the analysis in the section below.

Therefore, the hybrid block method is:

$$y_{n+1} = y_n + \frac{h}{2268000}\begin{bmatrix} -22823f_{n+5} + 1018705f_{n+4} - 5401280f_{n+7/2} + 14066950f_{n+3} - 22258304f_{n+5/2} + \\ 22802950f_{n+2} - 15231680f_{n+3/2} + 6764305f_{n+1} + 529177f_n \end{bmatrix}$$

Order P =9, $C_{10} = \frac{23}{113400}$

$$y_{n+3/2} = y_n + \frac{h}{3584000}\begin{bmatrix} -35451f_{n+5} + 1580535f_{n+4} - 8372160f_{n+7/2} + 21771650f_{n+3} - 34357248f_{n+5/2} + \\ 34977150f_{n+2} - 22327360f_{n+3/2} + 11304585f_{n+1} + 834299f_n \end{bmatrix}$$

Order P =9, $C_{10} = \frac{211638773}{36700160}$

$$y_{n+2} = y_n + \frac{h}{141750}\begin{bmatrix} -1411f_{n+5} + 62960f_{n+4} - 333760f_{n+7/2} + 869150f_{n+3} - 1376128f_{n+5/2} + \\ 1436150f_{n+2} - 852160f_{n+3/2} + 445685f_{n+1} + 33014f_n \end{bmatrix}$$

Order P =9, $C_{10} = \frac{9}{44800}$

$$y_{n+5/2} = y_n + \frac{h}{2322432}\begin{bmatrix} -23015f_{n+5} + 1025875f_{n+4} - 5432000f_{n+7/2} + 14106250f_{n+3} - 21831680f_{n+5/2} + \\ 24133750f_{n+2} - 14024000f_{n+3/2} + 7310125f_{n+1} + 540775f_n \end{bmatrix}$$

Order P =9, $C_{10} = \frac{119125}{594542592}$

$$y_{n+3} = y_n + \frac{h}{28000}\begin{bmatrix} -279 f_{n+5} + 12465 f_{n+4} - 66240 f_{n+7/2} + 177350 f_{n+3} - 254592 f_{n+5/2} + \\ 289350 f_{n+2} - 168640 f_{n+3/2} + 88065 f_{n+1} + 6521 f_n \end{bmatrix}$$

Order P=9, $C_{10} = \frac{9}{44800}$

$$y_{n+7/2} = y_n + \frac{h}{41472000}\begin{bmatrix} -408317 f_{n+5} + 18046945 f_{n+4} - 89069120 f_{n+7/2} + 278121550 f_{n+3} - 382140416 f_{n+5/2} + \\ 430928050 f_{n+2} - 250550720 f_{n+3/2} + 130568095 f_{n+1} + 9655933 f_n \end{bmatrix}$$

Order P =9, $C_{10} = \frac{5243}{26214400}$

$$y_{n+4} = y_n + \frac{h}{70875}\begin{bmatrix} -736 f_{n+5} + 43010 f_{n+4} - 117760 f_{n+7/2} + 454400 f_{n+3} - 636928 f_{n+5/2} + \\ 727400 f_{n+2} - 424960 f_{n+3/2} + 222560 f_{n+1} + 16514 f_n \end{bmatrix}$$

Order P =9, $C_{10} = \frac{-77459101}{1607445000}$

$$y_{n+5} = y_n + \frac{h}{18144}\begin{bmatrix} 4045 f_{n+5} + 65125 f_{n+4} - 152000 f_{n+7/2} + 298750 f_{n+3} - 431120 f_{n+5/2} + \\ 298750 f_{n+2} - 152000 f_{n+3/2} + 65125 f_{n+1} + 4045 f_n \end{bmatrix}$$

Order P =9, $C_{10} = -\frac{1865576447}{193133445}$

(13)

4. ANALYSIS OF THE NEW METHODS

4.1 Consistence

The block method (13) is consistent since it has order $p = 9 \geq 1$.

4.2 Zero Stability

Definition 4.1

For *n* = *mr* for some integer m ≥ 0, a block method is said to be zero stable if the roots $R_j, j = 1,...,k$ of the first characteristic polynomial

$\rho(R) = \det\left[\sum_{j=0}^{k} A^{'} R^{k-i}\right] = 0$ Satisfies $\left|R_j\right| \leq 1$ and for those roots with $\left|R_j\right|$=1, the multiplicity must not exceed one. Thus,

$$\rho(R) = \det\left[RA^{0} - A^{'}\right]$$

$$\rho(R) = \det\left[R\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&1\end{pmatrix} - \begin{pmatrix}0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&0&1\end{pmatrix}\right] = R^{7}(R-1) \tag{14}$$

R=0, R=1 and by definition (4.1), the block hybrid method (13) is zero stable.

4.3 Convergence

The block method (13) is convergent by consequence of Dahlquist theorem below.

***Theorem* 4.1:** *The necessary and sufficient conditions that a continuous LMM be convergent are that it be consistent and zero-stable.*

4.4 Region of Absolute Stability

The stability polynomial for our method is given by

$$-h^{8}\left(\left(\frac{19135}{497664}\right)w^{7} - \left(\frac{125305}{1492992}\right)w^{8}\right) - h^{7}\left(\left(\frac{7480219}{2090188}\right)w^{8} + \left(\frac{2321393}{6967296}\right)w^{7}\right)$$
$$-h^{6}\left(\left(\frac{473735837}{627056640}\right)w^{8} + \left(\frac{56612707}{69672960}\right)w^{7}\right) + h^{5}\left(\left(\frac{523261}{221184}\right)w^{7} - \left(\frac{20689721}{17915904}\right)w^{8}\right)$$
$$+h^{4}\left(\left(\frac{294864509}{26873856}\right)w^{8} + \left(\frac{45424367}{2322432}\right)w^{7}\right) + h^{3}\left(\left(\frac{180861131}{4478976}\right)w^{8} + \left(\frac{7860527}{165888}\right)w^{7}\right)$$
$$+h^{2}\left(\left(\frac{9677381}{209952}\right)w^{8} + \left(\frac{84377}{2016}\right)w^{7}\right) - h\left(\left(\frac{5}{2}\right)w^{8} + \left(\frac{5}{2}\right)w^{7}\right) + w^{8} - w^{7}$$

Using MATLAB software, the absolute stability region of the new method is plotted and shown in Fig. 1.

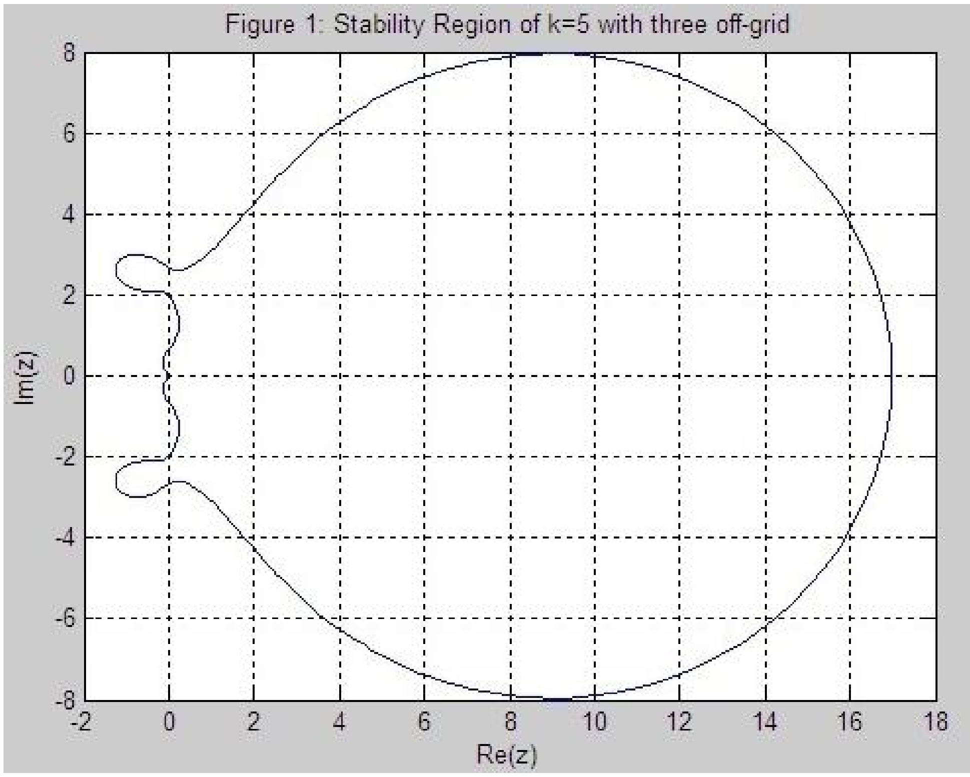

Fig. 1. Stability region of k=5 with three off-grid

According to Lambert (1973), the stability region for L-stable schemes must encroach into the positive half of the complex z plane

Note: the off-step points $x_{n+\frac{3}{2}}$, $x_{n+\frac{5}{2}}$, $x_{n+\frac{7}{2}}$ are used as an additional collocation points to the earlier known ones x_n, x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4} and x_{n+5}. This shows an L-Stable block hybrid method of order eight.

5. NUMERICAL EXPERIMENTS

The newly constructed method in equation (13) is tested on both mildly stiff and non-stiff IVP problems and the results are displayed in Table 1 and Table 2 below.

The following notations are used in the tables below.

ERR-Exact Solution-Computed Result
ESSI-Error in Skwame et al. (2012)
EMY-Error in Mohammed and Yahaya [8]

5.1 Problem 5.0.1

Consider the mildly stiff IVP,

$y' = \lambda y$, $y(0) = 1$, $h = 0.01$, $\lambda = -10$

Exact solutions: $y(x) = e^{\lambda x}$

[7] solved this problem. They solved the problem by adopting as L-stable hybrid block Simpson's method of order six.

5.2 Problem 5.0.2

Consider the non-stiff IVP,

$y' = -y$, $y(0) = 1$,

Exact solutions: $y(x) = e^{-x}$

[8] Solved this problem, by adopting fully implicit four points block backward formula.

Table 1. Showing the result for mildly stiff problem 5.0.1

x	Exact solution	Computed solution	ERR	ESSI
0.01	0.9048374180359595	0.9048374180555556	1.959610e-011	6.28e-03
0.02	0.8187307530779818	0.8187307531134442	3.546241e-011	1.88e-03
0.03	0.7408182206817179	0.7408182207298494	4.813150e-011	3.26e-03
0.04	0.6703200460356393	0.6703200460937075	5.806822e-011	1.06e-02
0.05	0.6065306597126334	0.6065306597783113	6.567791e-011	3.85e-03
0.06	0.5488116360940264	0.5488116361653398	7.131340e-011	1.45e-03
0.07	0.4965853037914095	0.4965853038666910	7.528156e-011	5.02e-04
0.08	0.4493289641172216	0.4493289641950702	7.784862e-011	2.76e-04
0.09	0.4065696597405992	0.4065696598198445	7.924533e-011	1.01e-04
0.10	0.3678794411714423	0.3678794412511137	7.967133e-011	3.74e-05

Table 2. Showing the result for non stiff problem 5.0.2

x	Exact solution	Computed solution	ERR	EMY
0.01	0.9048374180359595	0.9048374180555556	1.959610e-011	2.5292e-06
0.02	0.8187307530779818	0.8187307531134442	3.546241e-011	2.0937e-06
0.03	0.7408182206817179	0.7408182207298494	4.813150e-011	2.0079e-06
0.04	0.6703200460356393	0.6703200460937075	5.806822e-011	1.6198e-06
0.05	0.6065306597126334	0.6065306597783113	6.567791e-011	3.1608e-06
0.06	0.5488116360940264	0.5488116361653398	7.131340e-011	2.7294e-06
0.07	0.4965853037914095	0.4965853038666910	7.528156e-011	2.5457e-06
0.08	0.4493289641172216	0.4493289641950702	7.784862e-011	2.1713e-06
0.09	0.4065696597405992	0.4065696598198445	7.924533e-011	3.1008e-06
0.10	0.3678794411714423	0.3678794412511137	7.967133e-011	2.7182e-06

6. DISCUSSION OF THE RESULTS

We have considered two numerical examples to test the efficiency of our method. [7] solved problem 5.1. They adopted an L-Stable hybrid block Simpsons' method of order six. The new method gave a better approximation because the proposed method is self-starting and does not require starting values. [8] solved problem 5.2. They proposed a fully implicit four-point block backward difference formula. Our new method also gave better approximation.

7. CONCLUSION

We proposed an order nine continuous hybrid block method for the solution of first order ordinary differential equations. Our method was found to be zero stable, consistent and convergent. The numerical examples considered showed that our method gave better accuracy than the existing methods.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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